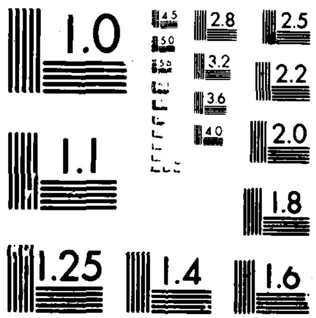
AD-A081 439 CARNEGIE-MELLON UNIV PITTSBURGH PA DEPT OF COMPUTER --ETC F/G 12/1
ANNOTATED BIBLIOGRAPHY AND BRIEF HISTORY OF OPTIMAL ALGORITHMS --ETC(U)
AUG 79 J F TRAUB, H WOZNIAKOWSKI N00014-76-C-0370
UNCLASSIFIED CMU-CS-79-140 NL

1 OF 2
AD
A081439


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

ANNOTATED BIBLIOGRAPHY AND BRIEF HISTORY OF OPTIMAL ALGORITHMS AND ANALYTIC COMPLEXITY

J. F. Traub
Department of Computer Science
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213

H. Woźniakowski
Department of Computer Science
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213
(Visiting from the University of Warsaw)

August 1979

DEPARTMENT
of
COMPUTER SCIENCE

Carnegie-Mellon University

(14) CMU-CS-79-140

(6) (12)

ANNOTATED BIBLIOGRAPHY AND BRIEF HISTORY OF OPTIMAL ALGORITHMS AND ANALYTIC COMPLEXITY

(10) J. F. Traub
Department of Computer Science
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213

H. Wozniakowski
Department of Computer Science
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213
(Visiting from the University of Warsaw)

(9) Interim rept.

(11) August 1979 (12) 172

(15) N00014-76-C-0370
NSF-MCS78-23676

This research was supported in part by the National Science Foundation under Grant MCS-7823676 and the Office of Naval Research under Contract N00014-76-C-0370, NR 044-422.

403081 LB

Ø.1

ABSTRACT

This is an annotated bibliography of over 300 papers and books on optimal algorithms and analytic complexity covering both the eastern European and the western literature. Each bibliographic item consists of a bibliographic reference, a set of keywords, and a short description. A brief history of the subject is also included.

This annotated bibliography will appear in a monograph, <u>A General Theory of Optimal Algorithms</u>, Academic Press, 1980.

1. INTRODUCTION

This annotated bibliography of over 300 papers and books on optimal algorithms and analytic complexity covers both the eastern European and the western literature. We also include a very brief history of the subject. Each bibliographic item consists of a bibliographic reference, a set of keywords, and a short description.

To keep the bibliography of manageable size we've limited ourselves almost entirely to items central to the subject; such items are assigned the keyword core. We've included a few exceptionally relevant mathematical works; these are assigned the keyword mathematics. Even in the core, we have been selective, choosing only what we regard as the most important works, a process which has necessitated some hard choices.

We have generally included only items which study problems which can only be solved approximately and omitted those dealing with problems which are solved approximately for reasons of efficiency. Thus we've omitted iterative solution of large linear systems and approximate solution of hard combinatorial problems (such as NP-complete problems); each of these has its own extensive literature. Also excluded is the huge literature on analysis of a specific algorithm since there is then no study of optimality. In particular, we've excluded the large literature on convergence, order, etc. of iterative algorithms for the solution of nonlinear equations. We've omitted most items dealing with mathematical theories even if germane to our subject. Examples include theories of approximation, differential equations, and iteration.

We've sometimes chosen to use in our descriptions unifying concepts and terminology only recently introduced rather than the terminology of the

author. Thus we use words such as information and complexity index.

Many different definitions of optimality appear in the literature. Often an optimal algorithm is defined as enjoying minimal error in some restricted class of algorithms, such as the class of linear algorithms. If the author is considering optimal algorithms in some restricted sense, then we use the phrase <u>optimal algorithm</u> in our keyword list and in our description. If the algorithm has minimal error in the class of all algorithms using the same information, then it is called an <u>optimal error algorithm</u>.

Many authors state their results in terms of n, which we call "cardinality of information". Although we have generally followed the author's usage, it would be routine to restate such results as a complexity function of ϵ.

We provide English titles for Russian and Polish papers which have not been translated into English. For translated papers we provide references to both the original and the translation and use the title provided by the translator. If a periodical supplies a translated title for an untranslated paper, we use that title.

Since the theory of optimal algorithms and analytic complexity is a rapidly evolving subject, we will prepare updated versions of this bibliography. The literature of this subject is large, diffuse, and since it is widely applicable, appears in many different periodicals. We solicit suggestions from our readers on important entries that we may have overlooked.

2. BRIEF HISTORY

The theory of optimal algorithms and analytic complexity has two major streams. One stream, which in our terminology studies general information, started with the work of Kiefer, Sard, and Nikolskij around 1950. The other stream, which studies iterative information, began with Traub in 1961. We indicate a very few of the major achievements in each of these two streams starting with the general information case. The annotated bibliography gives a more complete history.

Kiefer [53] showed that if function evaluations are used, then Fibonacci search is optimal in searching for the maximum of a unimodal function. Professor Kiefer has informed us that this work was done as an MIT Master's Thesis in 1948 but was only published later with the encouragement of J. Wolfowitz.

Sard [49] studied optimal algorithms for quadrature which use function evaluations at fixed points and discussed extending his results to the approximation of linear functionals. Independently, Nikolskij [50] posed the same problem and permitted the points of evaluation to be optimally chosen. Sard and Nikolskij restricted themselves to linear algorithms. In his dissertation Smolyak [65] proved that for any linear functional defined on a balanced convex set and for any information operator consisting of n linear functionals there exists a linear optimal error algorithm. Therefore, linear algorithms optimal in the sense of Sard or Nikolskij are optimal error algorithms, provided the set of elements is balanced and convex.

Golomb and Weinberger [59] performed the first systematic study of optimal error algorithms for approximation of a linear functional with information consisting of n linear functionals.

For many problems, optimal error algorithms are based on interpolatory splines. Schoenberg [64] was the first to recognize the close connection between splines and optimal algorithms in the sense of Sard.

Winograd [76] discussed using adversary arguments to obtain lower bounds in analytic complexity.

Micchelli and Rivlin [77] studied optimal algorithms for linear problems using linear information.

Traub and Woźniakowski [77] made the concept of information basic. This model permits linear and nonlinear problems and operators. They introduce the concept of radius (diameter) of information in a general setting and used it to obtain a very powerful adversary principle. This initiated the study of lower bounds on complexity in a general setting. The notion of central algorithm was introduced. Some of these concepts had been implicitly used in special cases in a number of earlier papers.

One of the basic problems of analytic complexity is to find the most relevant information for a given problem. This was first studied for specific problems by Nikolskij [50] and Kiefer [53]. The idea of varying the information operator, which leads to the concept of optimal information operators was introduced in Traub and Woźniakowski [77].

The second major stream of research in analytic complexity, which studied iterative information and iterative algorithms, had its inception in the work of Traub [61, 64]. Iterative algorithms were classified by the

information they use. Theorems were obtained and conjectures proposed on maximal order of iterative algorithms for solving scalar nonlinear equations. Such maximal order results are needed to obtain lower bounds on complexity. The term "analytic computational complexity" was coined by Traub [72] although the work described in that paper is restricted to the portion of analytic complexity which would now be called iterative computational complexity.

Brent, Winograd, and Wolfe [73] used an adversary argument to obtain a maximal order theorem for the case of a class of nonstationary one-point iterations with memory using standard information to solve scalar nonlinear equations. Woźniakowski [75] introduced the concept of order of information which provides a powerful tool for establishing maximal order in an abstract space. He showed that maximal order in a class of algorithms depends only on the information used by an algorithm and not on the structure of the algorithm.

Traub and Woźniakowski [76] posed a new question: what information is relevant to the solution of a problem? A complete answer is provided for one-point iterations with linear information.

Traub and Woźniakowski [78] obtained a powerful adversary principle for establishing lower bounds on complexity in the iterative information model.

The two long reports Traub and Woźniakowski [77, 78] for the first time bring together the two streams by including both in the same abstract setting and noting that they differ principally in whether general or iterative information is used. Traub and Woźniakowski [80] is a monograph which includes considerable extended and improved material from these two reports. It also includes this bibliography.

Brent, Winograd and Wolfe [73]	Brent, R. P., Winograd, S. and Wolfe, P., "Optimal Iterative Processes for Root-finding," Numer. Math., 20, 1973, 327-341.
Golomb and Weinberger [59]	Golomb, M. and Weinberger, H. F., "Optimal Approximation and Error Bounds," in On Numerical Approximation, edited by R. E. Langer, The University of Wisconsin Press, Madison, 1959, 117-190.
Kiefer [53]	Kiefer, J., "Sequential Minimax Search for a Maximum," Proc. Amer. Math. Soc., 4, 1953, 502-505.
Micchelli and Rivlin [77]	Micchelli, C. A. and Rivlin, T. J., "A Survey of Optimal Recovery," in Optimal Estimation in Approximation Theory, edited by C. A. Micchelli and T. J. Rivlin, Plenum Press, New York, 1977, 1-54.
Nikolskij [50]	Nikolskij, S. M., "On the Problem of Approximation Estimate by Quadrature Formulae," Uspekhi. Matem. Nauk, 5, 1950, 165-177.
Sard [49]	Sard, A., "Best Approximate Integration Formulas; Best Approximation Formulas," Amer. J. Math., 71, 1949, 80-91.
Schoenberg [64]	Schoenberg, I. J., "Spline Interpolation and Best Quadrature Formulae," Bull. Amer. Soc., 70, 1964, 143-148.
Smolyak [65]	Smolyak, S. A., "On an Optimal Restoration of Functions and Functionals of Them," (in Russian), Candidate Dissertation, Moscow State University, 1965.
Traub [61]	Traub, J. F., "On Functional Iteration and Calculation of Roots," Preprints of Papers 16 National ACM Conference, Session 5A-1, Los Angeles, CA, 1961, 1-4.
Traub [64]	Traub, J. F., Iterative Methods for Solution of Equations, Prentice-Hall, Englewood Cliffs, NJ, 1964.
Traub [72]	Traub, J. F., "Computational Complexity of Iterative Processes," SIAM J. Comput., 1, 1972, 167-179.

Traub and Woźniakowski [76]	Traub, J. F. and Woźniakowski, H., "Optimal Linear Information for the Solution of Nonlinear Equations," in Algorithms and Complexity: New Directions and Recent Results, edited by J. F. Traub, Academic Press, New York, 1976, 103-119.
Traub and Woźniakowski [77]	Traub, J. F. and Woźniakowski, H., "General Theory of Optimal Error Algorithms and Analytic Complexity, Part A: General Information Model," Department of Computer Science Report, Carnegie-Mellon University, 1977.
Traub and Woźniakowski [78]	Traub, J. F. and Woźniakowski, H., "General Theory of Optimal Error Algorithms and Analytic Complexity, Part B: Iterative Information Model," Department of Computer Science Report, Carnegie-Mellon University, 1978.
Traub and Woźniakowski [80]	Traub, J. F. and Woźniakowski, H., A General Theory of Optimal Algorithms, Academic Press, New York, 1980.
Winograd [76]	Winograd, S., "Some Remarks on Proof Techniques in Analytic Complexity," in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 5-15.
Woźniakowski [75]	Woźniakowski, H., "Generalized Information and Maximal Order of Information for Operator Equations," SIAM J. Num. Anal., 12, 1975, 121-135.

3. ANNOTATED BIBLIOGRAPHY

Adamski, A., Korytowski, A. and Mitkowski, W.,
"A Conception of Optimality for Algorithms and its Application to the Optimal Search for a Minimum,"
Zastosowania Matematyki, 14, 1977, 499-509.

core, extremum, scalar, optimal error algorithms

Considers the concept of "strong" optimal algorithms and tests this concept on the problem of searching for the minimum of a unimodal or convex scalar function. The information is the values of f at n points. Results are related to those of Kiefer [53].

Ahlberg, J. H. and Nilson, E. N.,
"The Approximation of Linear Functionals,"
SIAM J. Numer. Anal., 3, 1966, 173-182.

core, approximation of linear functionals

Considers approximation of a linear functional L for a class of scalar functions f such that $f^{(n)} \in L_2$. The information is the values of f and its derivatives. Shows that the value of L on an interpolatory spline minimizes a certain type of error.

Ahlberg, J. H., Nilson, E. N. and Walsh, J. L.,
The Theory of Splines and Their Applications,
Academic Press, New York , 1967.

mathematics and core, splines

Considers the theory of splines and their applications for a variety of problems in numerical analysis. Optimal properties of splines are studied. Shows the relationship between splines and optimal approximation in the sense of Sard.

Aksen, M. B. and Tureckij, A. H.,
"Best Quadrature Formulas for Certain Classes of Functions," (in Russian),
Dokl. Akad. Nauk SSSR, 166, 1966, 1019-1021. English translation in
Soviet Math. Dokl., 7, 1966, 203-205.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar functions with bounded r-th derivative in L_q. The information is the values of $f, f', \ldots, f^{(r-2)}$ at m points. The errors of optimal linear algorithms with optimally chosen points of information are derived for even r.

Alhimova, V. M.,

"Best Quadrature Formulas with Equidistant Nodes," (in Russian),

Dokl. Akad. Nauk SSSR,204, 1972, 263-266. English translation in Soviet Math. Dokl., 13, 1972, 619-623.

core, integration, scalar, optimal linear algorithms

Considers integration for some classes of scalar functions with bounded r-th derivative in L_q. The information is the values of f. Presents optimal quadrature formulae with equidistant points of information. The errors of such formulae are also given.

Aliev, R.M.: See Ibragimov, I. I.

Aphanasjev, A. Yu.,

"On the Search of Minimum Function with Limited Second Derivative,"

(in Russian),

Zh. vychisl. Mat. mat. Fiz., 14, 1974, 1018-1021. English translation, Afanas'ev, A. Yu., "The Search for the Minimum of a Function with a Bounded Second Derivative," U.S.S.R. Computational Math. and Math. Phys., 14, 1974, 191-195.

core, extremum, scalar, optimal error algorithms

Considers the search for the minimum in the class of scalar functions f whose second derivative belongs to [a,b] with a > 0. The information is the values of f at two points. The interval where a minimum of f lies is derived. Optimal points of information are obtained.

Aphanasjev, A. Yu. and Novikov, V. A.,

"On the Search of Minimum of a Function with the Limited Third Derivative," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 17, 1977, 1031-1034.

core, extremum, scalar, optimal error algorithms

Considers the search for the minimum in the class of scalar unimodal functions whose third derivative lies in a given interval. The information is the values of f at three points. An algorithm that finds an interval where a minimum of f lies is proposed.

Arestov, V. V.,

"On the Best Approximation of Differentiation Operators," (in Russian),

Mathematicheskie Zametki, 1, 1967, 149-154.

English translation in Math. Notes, 1, 1967, 100-103.

core, differentiation, optimal approximation by bounded linear operators

Continuation of Stechkin [67]. Considers approximation of $f^{(k)}$ for the class of scalar functions with bounded n-th derivative in L_1 or C, $0 < k < n$, by means of linear operators φ whose norm is bounded by a given constant. For small n shows the nearly optimal operator φ requires a few evaluations of f.

Arestov, V. V.,

"On the Best Uniform Approximation of Differentiation Operators," (in Russian),

Mathematicheski Zametki, 5, 1969, 273-284.

English translation in Math. Notes, 5, 1969, 167-173.

core, differentiation, optimal approximation by bounded linear operators

Continuation of Arestov [67]. Studies the existence and uniqueness of optimal linear operators for the approximation of $f^{(k)}$.

Arro, V. K.: See Levin, M. I.

Aubin, J. P.,

"Best Approximation of Linear Operators in Hilbert Spaces,"

SIAM J. Numer. Anal., 5, 1968, 518-521.

core, approximation of linear operators

Considers approximation of a linear operator $A : E \to F$ for the unit ball, E, F are Hilbert spaces. The information operator is a given linear operator $r_n : E \to E_n$. For a given linear operator $s_n : F \to F_n$, the operator A is approximated by a linear operator $A_n : E_n \to F_n$ such that $\| A_n r_n u - s_n Au \|_{F_n}$ for $\| u \|_E \leq 1$ is minimized. The solution is obtained in terms of the operator of "best interpolation".

Avriel, M. and Wilde, D. J.,

"Optimal Search for a Maximum with Sequences of Simultaneous Function Evaluations,"

Manag. Science, 12, 1966, 722-731.

core, extremum, scalar, optimal algorithms

Considers the search for the maximum in a class of scalar unimodal functions. The information is the values of f. Optimal algorithms based on simultaneous function evaluations are studied.

Babenko, V. F.,

"Asymptotically Sharp Bounds for the Remainder for the Best Quadrature Formulas for Several Classes of Functions," (in Russian),

Matematicheski Zametki, 19, 1976, 313-322.

English translation in Math. Notes, 19, 1976, 187-193.

core, integration, multivariate, asymptotic error bounds

Considers cubature formulae for the class of scalar functions of several variables with bounded modulus of continuity. The information is the values of f at n points. The asymptotic error for the optimal cubature formula (i.e., for large n) is derived.

Babuška, I.

"Problems of Optimization and Numerical Stability in Computations,"

Aplikace Matematiky, 1, 1968, 3-26.

core, integration, scalar, optimal algorithms, numerical stability

This is a paper presented at the conference "Basic problems of numerical mathematics" in Liblice 1967. Among the problems considered is integration for a class of periodic functions. The information is the values of f. Optimal points of information and optimal algorithms are studied. See also Babuška, I., "Über Universal Optimale Quadraturformeln" in Aplikace Matematiky, 4, 1968, 305-338 and 5, 1968, 388-404.

Babuška, I. and Sobolev, S. L.,

"Optimization of Numerical Methods," (in Russian),

Aplikace Matematiky, 1, 1965, 96-130.

core, survey of optimal algorithms

Considers optimal numerical algorithms for the solution of algebraic and analytic problems. Optimal or asymptotically optimal error algorithms for the approximation of linear functionals and linear equations with compact inverse operators are studied. Relations to Kolmogorov n-widths and entropy are given. Good bibliography.

Bakhvalov, N. S.,

"On the Approximate Calculation of Multiple Integrals," (in Russian), Vestn. MGU. Ser. of Math. Mech. Astron. Phys. Chem., 4, 1959, 3-18.

core, integration, multivariate, optimal linear algorithms, lower bounds

Considers integration for the class of scalar functions of s variables with all derivatives up to order p bounded and the p-th derivative satisfying the Hölder condition of order λ. The information is the values of f. Shows that a lower bound on the error of any quadrature formula with n points is roughly $n^{-(p+\lambda)/s}$. The expected value of the error has lower bound roughly $n^{-(p+\lambda)/s-.5}$. The bounds are shown to be sharp.

Bakhvalov, N. S.,

"An Estimate of the Mean Remainder in Quadrature Formulae," (in Russian), Zh. vychisl. Mat. mat. Fiz., 1, 1961, 64-77.

English translation in U.S.R.R. Computational Math. and Math. Phys., 1, 1961, 68-82.

core, integration, multivariate, average case analysis

This is a continuation of Bakhvalov [59]. For the same classes of functions, derives the expected values of the errors in quadrature formulae with f evaluated at random points.

Bakhvalov, N. S.,

"On Optimal Methods of Specifying Information in the Solution of Differential Equations," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 2, 1962, 569-592.

English translation in U.S.S.R. Computational Math. and Math. Phys., 2, 1962, 608-640.

core, differential equations, entropy, n-widths

Considers the differential equation $u_t = P(t,x,u)$ where $u = u(t,x)$ and P is a given operator, for a class of functions $u(0,x)$. The information is the values of $u(0,x)$. Studies the problem of the minimal number of evaluations of $u(0,x)$ to approximate $u(t,x)$ to within ε. Shows that this problem is related to ε-entropy. Also considers the approximation of a function from its n values by means of algorithms whose range has dimension n. Shows the relation to Kolmogorov n-widths.

Bakhvalov, N. S.,

"On the Estimate of the Amount of Computational Labor Necessary in Approximate Solutions," (in Russian),

Appendix IV in the book of S. K. Godunov, and W. S. Riabenki, Theory of Difference Schemes - An Introduction, Moscow, 1962, 316-329.

English translation of the book published by American Elsevier Publishing Company, 1964, 268-279.

core, integration, multivariate, differential equations, lower and upper bounds

Considers the complexity of integration for multivariate functions and the complexity of certain types of differential equations. The information is the values of f at n points. Lower and upper bounds on the complexity are derived.

Bakhvalov, N. S.,

"Optimal Properties of Adams and Gregory Formulae of Numerical Integration,"

(in Russian),

in Problems of Computational Mathematics and Computational Technique,

edited by L. A. Ljusternik, Mashgiz, Moscow, 1963, 9-26.

core, integration, scalar, differential equations, optimal algorithms

Considers integration for the class of scalar functions with bounded r-th derivative. The information is the values of f and its derivatives at equidistant points. Shows that the Euler and Gregory formulae are nearly optimal. Considers also ordinary differential equations, $y' = f(x,y)$, for a class of functions f, and shows that the Adams formulae are asymptotically optimal.

Bakhvalov, N. S.,

"On Optimal Bounds for the Convergence of Quadrature Formulas and Mont-Carlo Type Integration Methods for Classes of Functions," (in Russian),

in Numerical Methods for the Solution of Differential and Integral Equations and Quadrature Formulas, Moscow, 1964, 5-63.

core, integration, multivariate, asymptotically optimal algorithms, lower bounds

Considers integration for many classes of scalar functions of several variables. The information is the values of f at n points. Presents linear algorithms whose errors differ from the optimal error by at most a factor of $\ln^{\gamma} n$ for a positive γ.

Bakhvalov, N. S.,

"On the Optimal Speed of Integrating Analytic Functions," (in Russian), Zh. vychisl. Mat. mat. Fiz., 7, 1967, 1011-1020.

English translation in U.S.S.R. Computational Math. and Math. Phys., 7, 1967, 63-75.

core, integration, analytic functions, scalar, optimal algorithms

Considers integration for the class of analytic scalar functions bounded by a constant on the ellipse with foci ± 1 and the sum of semi-axes equal to a given c. The information is the values of f and f' at n points. Asymptotic optimality of Gauss quadrature is proven. The optimal error is roughly c^{-2n}.

Bakhvalov, N. S.,

"On Optimal Methods for the Solution of Problems," (in Russian), Aplikace Matematiky, 1, 1968, 27-38.

core, survey of optimal algorithms

This is a paper presented at the conference "Basic Problems of Numerical Mathematics" in Liblice 1967. Surveys Russian work on the optimal solution of many numerical problems.

Bakhvalov, N. S.,

"Properties of Optimal Methods for the Solution of Problems of Mathematical Physics," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 10, 1970, 555-568.

English translation in U.S.S.R. Computational Math. and Math. Phys., 10, 1970, 1-20.

core, integration, multivariate, differential equations, lower bounds

Considers optimal methods for solving problems of mathematical physics. The information is the values of f at n points. The minimum number of function evaluations to solve the problem to within ϵ is studied for multivariate integration and for parabolic differential equations. An asymptotically optimal algorithm with linear combinatory complexity is proposed for parabolic differential equations.

Bakhvalov, N. S.,

"On the Optimality of Linear Methods for Operator Approximation in Convex Classes of Functions," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 11, 1971, 1014-1018.

English translation in U.S.S.R. Computational Math. and Math. Phys., 11, 1971, 244-249.

core, approximation of linear functionals, optimal linear algorithms

Considers approximation of linear functionals for a balanced convex class. The information is the values of n linear functionals. Contains Smolyak's lemma which states that there exists a linear optimal error algorithm. Some extensions to the approximation of linear operators are presented.

Bakhvalov, N. S.,
"Optimization of Methods of Solving Ordinary Differential Equations with Strongly Oscillating Solutions," (in Russian),
Zh. vychisl. Mat. mat. Fiz., 11, 1971, 1318-1322.
English translation in U.S.S.R. Computational Math. and Math. Phys., 11, 1971, 287-292.

core, differential equations, lower bounds

Considers the equation $\mu^2 y'' + a(x)y = f(x)$ for small positive μ for the class of functions such that $a(x) \geq a_0 > 0$, $|f^{(i)}(x)| \leq A$ for $i = 0,1,\ldots,m$, $\forall x \in [0,1]$ and $|\mu y'(0)| \leq b_0$. The information is the values of a and f. Shows that a lower bound on any algorithm using n evaluations of a has error at least roughly $\min(1,1/(\mu n^m))$. For m = 1 or 2 this bound is sharp.

Bakhvalov, N. S.,
"A Lower Bound for the Asymptotic Characteristics of Classes of Functions with Dominating Mixed Derivative," (in Russian),
Mathematicheskie Zametki, 12, 1972, 655-664.
English translation in Math. Notes, 12, 1972, 833-838.

core, integration, interpolation, periodic functions, multivariate, lower bounds

Considers integration and interpolation for the class of scalar periodic functions of several variables with bounded derivative. The information is the values of f and its derivatives. Using a new representation of the class of functions, lower bounds for the errors in integration and interpolation, and a lower bound for the ϵ-entropy are given.

Barnhill, R. E.,

"Optimal Quadratures in $L^2(E_\zeta)$. I and II,"

<u>SIAM J. Numer. Anal.</u>, 4, 1967, 390-397 and 534-541.

core, integration, analytic functions, scalar, optimal linear algorithms

Considers integration for the class of scalar analytic functions bounded by a constant in the ellipse E_ζ with foci ± 1, semi-axes a, b and $\zeta = (a+b)^2$. The information is the values of f. Optimal algorithms are derived.

Barnhill, R. E.,

"Asymptotic Properties of Minimum Norm and Optimal Quadratures,"

<u>Numer. Math.</u>, 12, 1968, 384-393.

core, integration, analytic functions, scalar, optimal linear algorithms, Gauss quadrature

Considers integration for the class of scalar analytic functions on the ellipse E_ζ with foci at ± 1, semi-axes a,b with $\zeta = (a+b)^2$. The information is the values of f. Asymptotic properties of optimal quadrature formulae are studied. Shows that the weights and points of an optimal quadrature converge to the weights and points of Gauss quadrature as $\zeta \to +\infty$.

Barnhill, R. E. and Wixom, J. A.,

"Quadratures with Remainders of Minimum Norm. I and II,"

Math. Comp., 21, 1967, 66-75 and 382-387.

core, integration, analytic functions, scalar, optimal points of information, optimal linear algorithms

Considers integration for a class of scalar analytic functions defined on the ellipse E_ζ with foci ± 1, semi-axes a,b and $\zeta = (a+b)^2$. The information is the values of f. Optimal quadrature formulae for fixed and varying points of information are discussed.

Barnhill, R. E. and Wixom, J. A.,

"An Error Analysis for Interpolation of Analytic Functions,"

SIAM J. Numer. Anal., 5, 1968, 522-528.

core, interpolation, analytic functions, scalar, optimal linear algorithms

Considers interpolation for a class of scalar analytic functions defined on the ellipse E_ζ with foci ± 1, semi-axes a,b and $\zeta = (a+b)^2$. The information is the values of f. Linear optimal algorithms are studied. Asymptotic properties of the optimal weights are considered as $\zeta \to +\infty$.

Barrar, R. B., Loeb, H. L. and Werner, M.,

"On the Existence of Optimal Integration Formulas for Analytic Functions,"

Numer. Math., 23, 1974, 105-117.

core, integration, scalar, optimal linear algorithms

Considers integration for a class of analytic functions. The information is the values of f and its derivatives. The existence of weights and points of a quadrature formula which minimizes a certain error is proven.

Baudet, G. M.,

"Asynchronous Iterative Methods for Multiprocessors,"

J.ACM, 25, 1978, 226-244.

core, nonlinear equations, multivariate, iterative algorithms, asynchronous

Introduces and analyzes a general class of asynchronous iterative methods. Establishes general convergence theorem and obtains complexity bounds. Presents experimental results for certain problems which show that "purely asynchronous" iterative methods are best.

Beamer, J. H. and Wilde, D. J.,

"Time Delay in Minimax Optimization of Unimodal Functions of One Variable,"

Manag. Science, 15, 1969, 528-538.

core, extremum, scalar, optimal algorithms

Considers the search for the maximum in a class of scalar unimodal functions. The information is the values of f. Optimal algorithms are derived for two cases. In the first, each function evaluation is performed before the preceding result is known. In the second case each is performed before the two preceding results are known.

Beamer, J. H. and Wilde, D. J.,

"Minimax Optimation of Unimodal Functions by Variable Block Search,"

Manag. Science, 16, 1970, 529-541.

core, extremum, scalar, optimal algorithms

Considers the search for the maximum in a class of scalar unimodal functions. The information is the values of f. Optimal algorithms based on simultaneous function evaluations are studied.

Beamer, J. H. and Wilde, D. J.,

"Minimax Optimization of a Unimodal Function by Variable Block Derivative Search with Time Delay,"

J. Comb. Th., 10, 1971, 160-173.

core, extremum, scalar, optimal algorithms

Considers the search for the maximum in a class of scalar unimodal functions. The information is the values of the first derivative of f. The search algorithms use a sequence of blocks of simultaneous evaluations of f'. Optimal error algorithms are derived. A method of optimizing the number of evaluations per block is given.

Bojanov, B. D.,

"Optimal Rate of Integration and ε-Entropy of a Class of Analytic Functions," (in Russian),

Matematicheskie Zametki 14, 1973, 3-10.

English translation in Math. Notes, 19, 1973, 551-556.

core, integration, analytic functions, scalar, lower bounds

See Bojanov [74]. The ε-entropy of a class of analytic functions is derived.

Bojanov, B. D.,
"Best Quadrature Formula for a Certain Class of Analytic Functions,"
Zastosowania Matematyki, 14, 1974, 441-447.

core, integration, analytic functions, scalar, optimal linear algorithms, lower bounds

Considers optimal quadrature formulas for the class of real functions on [-1,1] which can be analytically extended to the unit disk and whose extension is bounded by unity. The information is the values of f, f' at n points. Using Smolyak's lemma the linear optimal error algorithm and its error are derived. For optimal points the error is roughly $\exp(-\pi\sqrt{n}/\sqrt{2})$.

Bojanov, B.D.,
"Best Methods of Interpolation for Certain Classes of Differentiable Functions," (in Russian),
Matematicheski Zametki, 17, 1975, 511-524.
English translation in Math. Notes, 17, 1975, 301-309.

core, interpolation, scalar, optimal linear algorithms

Considers the interpolation problem for the class of scalar functions with bounded r-th derivative in L_q. The information is the values of $f, f', \ldots, f^{(r-1)}$ at n points. The linear optimal error algorithm is derived and shown to be a spline. The optimal points of information are proven to be equidistant.

A slightly extended version of B. D. Bojanov, "Optimal Methods of Interpolation in $W^{(r)}L_q(M; a,b)$," in English, Comptes Rendus de l'Academie Bulgare des Sciences, 17, 1974, 885-888.

Bojanov, B. D.,

"Optimal Methods of Integration in the Class of Differentiable Functions,"

Zastosowania Matematyki, 15, 1976, 105-115.

core, integration, scalar, optimal linear algorithms, lower bounds

Considers optimal quadrature formulae for the class of r times piecewise continuously differentiable functions whose rth derivative in L_q is bounded by a constant. The information is the values of $f, f', \ldots, f^{(r-1)}$ at n points. Using Smolyak's lemma the linear optimal error algorithm and its error are derived. For optimal points the error is obtained.

Bojanov, B. D. and Chernogorov, V. G.,

"An Optimal Interpolation Formula,"

J. Approximation Theory, 20, 1977, 264-274.

core, interpolation, approximation, scalar, optimal linear algorithms

Considers approximation of a linear functional for a given class of functions. The information is the values of n linear functionals on f. Optimal error algorithms are studied. Linear optimal error algorithms are presented for the interpolation and approximation problems in the class of scalar functions whose second derivative is bounded in L_∞ by a constant.

de Boor, C.,

"Computational Aspects of Optimal Recovery,"

in Optimal Estimation in Approximation Theory, edited by C. A. Micchelli and T. J. Rivlin, Plenum Press, New York, 1977, 69-91.

core, approximation, perfect splines

Considers approximation for the class of scalar functions whose kth derivative is bounded in L_∞. The information is the values of f and its derivatives. Presents a Fortran subroutine for the construction of a perfect interpolatory spline which is the optimal error algorithm. Related to Micchelli, Rivlin and Winograd [76] and Gaffney and Powell [76].

Booth, R. S.,

"Location of Zeros of Derivatives,"

SIAM J. Appl. Math., 15, 1967, 1496-1501.

core, nonlinear equations, scalar, optimal points of information, error bounds

Considers the search for a zero α of the k-th derivative in the class of scalar functions for which $f^{(k)}$ changes sign only at α. The information is the values of f at n points. Studies the asymptotic character of the error of an optimal algorithm for optimally chosen points of information.

Booth, R.S.,

"Location of Zeros of Derivatives. II,"

<u>SIAM J. Appl. Math.</u>, 17, 1969, 409-415.

core, nonlinear equations, scalar, optimal points of information, error bounds

Continuation of Booth [67].

Borodin, A. and Munro, I.,

<u>The Computational Complexity of Algebraic and Numeric Problems</u>,

American Elsevier, New York, 1975.

core, algebraic numbers, optimal iterations, algebraic complexity, iterative complexity, maximal order

A text on algebraic complexity. Includes chapter on parallel processing in numeric computation. Of particular relevance to analytic complexity is a chapter on "The Complexity of Rational Iterations" which covers the Paterson-Kung theory of the complexity of iterations which approximate algebraic numbers. See also Paterson [72], Kung [72, 73].

Brent, R. P.,

"The Computational Complexity of Iterative Methods for Systems of Non-linear Equations,"

in Complexity of Computer Computations, edited by R. E. Miller and J. W. Thatcher, Plenum Press, New York, 1972, 61-71.

core, nonlinear equations, multivariate, iterations with memory, complexity index, maximal order

Compares complexity of classes of algorithms for solving the system of nonlinear equations $f = 0$. The information is the values of f. The classes of algorithms considered include multivariate polynomial interpolatory methods as well as two new classes.

Brent, R. P.,

Algorithms for Minimization Without Derivatives,

Prentice-Hall, 1973.

core, nonlinear equations, extremum, scalar, multivariate, order

A valuable monograph on algorithms and programs for computing zeros and extrema of scalar nonlinear functions and extrema of multivariate nonlinear functions. The information is values of the function. Contains much original material. Some discussion of optimality and complexity. Good bibliography.

Brent, R. P.,

"Some Efficient Algorithms for Solving Systems of Nonlinear Equations,"

SIAM J. Numer. Anal., 10, 1973, 327-344.

core, nonlinear equations, multivariate, iterative algorithms, secant iteration, iterative complexity, complexity index, order

Considers iterative algorithms for solving a multivariate nonlinear system $f = 0$. The information is the values of f. Introduces two new classes of algorithms and establishes their local convergence. Computes a complexity index for these algorithms and compares with known methods. Poses an open problem on the optimal complexity index.

Brent, R. P.,

Computer Solution of Nonlinear Equations,

Lecture Notes, Computer Science Department, Stanford University, 1975.

core, nonlinear equations, scalar, multivariate, one-point iterations, multipoint iterations, iterations with memory, iterative complexity

A book-length set of notes. Surveys iteration algorithms and iterative complexity for both scalar and multivariate nonlinear equations. Good bibliography.

Brent, R. P.,

"Some High-Order Zero-Finding Methods Using Almost Orthogonal Polynomials,"

J. Austral. Math. Soc., 19 (Series B), 1975, 1-29.

core, nonlinear equations, scalar, iterative information, multipoint iterations, complexity index

Considers iterative algorithms for computing a zero of a scalar nonlinear function f. The information is the values of $f, f', \ldots, f^{(m)}$, at one point and n values of $f^{(k)}$ at distinct points. For $m > 0$, $n \geq 0$, and k satisfying $m+1 \geq k > 0$, there are algorithms of order $m + 2n + 1$. To establish convergence, results are obtained on orthogonal and "almost orthogonal" polynomials. Discusses a complexity index for the iterations. Good bibliography.

Brent, R. P.,

"A Class of Optimal-Order Zero-Finding Methods Using Derivative Evaluations,"

in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 59-73.

core, nonlinear equations, scalar, iterative information, multipoint iterations, maximal order

Considers iterative algorithms for computing a zero of a scalar nonlinear function f. The information is one evaluation of f and n evaluations of f'. The points of evaluation are determined from certain orthogonal or "almost orthogonal" polynomials. These iterations are of maximal order.

Let $x'(t) = g(x)$. The above results are used to obtain an explicit nonlinear Runge-Kutta method of order 2n-1 which uses n evaluations of g.

See also Brent [75], "Some Higher-Order Zero-Finding Methods Using Almost Orthogonal Polynomials."

Brent, R. P.,

"Fast Multiple-Precision Evaluation of Elementary Functions,"

J.ACM, 23, 1976, 242-251.

core, approximation, scalar, multiple precision, fast algorithms

Shows that elementary functions can be evaluated with relative error of $\Theta(2^{-n})$ in $\Theta(M(n)\log n)$ operations where $M(n)$ is the number of single-precision operations required to multiply n-bit integers. Special cases include the evaluation of constants such as π, e, and e^{π}.

Brent, R. P.,

"Multiple-Precision Zero-Finding Methods and the Complexity of Elementary Function Evaluation,"

in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 151-176.

core, nonlinear equations, formal power series, scalar, fast algorithms, multiple precision

Introduces fast algorithms and analyzes their complexity for multi-precision computation of certain numbers, arithmetic operations, and functions. Among the computations discussed are reciprocation, square roots, zero-finding, evaluation of π, evaluation of elementary transcendental functions, and the solution of scalar equations. Fast algorithms for such formal power series operations as logarithm and powering are given.

Brent, R. P.,

"The Complexity of Multiple-Precision Arithmetic,"

in Complexity of Computational Problem Solving, edited by R. Anderssen and R. P. Brent, *University of Queensland Press*, 1976, 125-165.

core, approximation and nonlinear equations, scalar, multiple precision, fast algorithms, lower bounds, upper bounds, iterative complexity

Studies complexity of performing multiple-precision computations. Among the computations considered are arithmetic operations, and elementary function evaluations. Upper bounds and some lower bounds are obtained. Also compares complexities of various iterations for computing zeros of nonlinear scalar functions using variable-length multiple-precision arithmetic.

Brent, R. P. and Kung, H. T.,

"$O((N \log N)^{3/2})$ Algorithms for Composition and Reversion of Power Series,"

in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 217-225.

core, composition, reversion, formal power series, fast algorithms, algebraic complexity

First announcement of the result of the title. See Brent and Kung [78] "Fast Algorithms for Manipulating Formal Power Series," for proof.

Brent, R. P. and Kung, H. T.,
"Fast Algorithms for Composition and Reversion of Multivariate Power Series,"
in Proceedings of Conference on Theoretical Computer Science, University of Waterloo, Waterloo, Canada, 1977, 149-158.

core, composition, reversion, formal power series, multivariate, fast algorithms, algebraic complexity

Extends results of Brent and Kung [78] to the multivariate case. Shows that every reversion problem can be associated with a composition problem in the sense that if the composition problem can be solved fast so can the reversion problem. Presents fast algorithms for composition and reversion of power series which require substantially fewer operations than classical methods. The improvement increases as the number of variables increases.

Brent, R. P. and Kung, H. T.,
"Fast Algorithms for Manipulating Formal Series,"
J.ACM, 25, 1978, 581-595.

core, composition, reversion, differential equations, formal power series, fast algorithms, algebraic complexity

Gives algorithm for computing first N terms of composite of two power series in $O((N \log N)^{3/2})$ operations. Shows that the complexity of composition and reversion are asymptotically equivalent. Let MULT(N) be the minimal number of operations for computing the first N terms of the product of two polynomials. Proves that the evaluation of the reversion series truncated to N terms can be done in O(MULT(N)) operations. Shows that the first N terms of the power series solutions to many types of differential equations can be obtained in O(MULT(N)) operations.

Brent, R. P. and Traub, J. F.,

"On the Complexity of Composition and Generalized Composition of Power Series,"

Computer Science Department Report, Carnegie-Mellon University, 1978. To appear in SIAM J. Comput.

core, composition, generalized composition, formal power series, fast algorithms, algebraic complexity

Let $F^{[q]}(x)$ be the qth composite of a formal power series. Shows that $F^{[q]}(x)$ can often, but not always, be defined for general q. Gives fast algorithms and complexity bounds for computing the first N terms of $F^{[q]}(x)$ whenever it is defined. If q is an integer, the fast algorithms eliminate the complexity factor of $\log_2 q$ of the "repeated squaring" algorithm.

Brent, R. P., Winograd, S. and Wolfe, P.,

"Optimal Iterative Processes for Root-Finding,"

Numer. Math., 20, 1973, 327-341.

core, nonlinear equations, scalar, iterative algorithms, iterations with memory, maximal order

Considers locally convergent nonstationary one-point iterations with memory for computing a zero of a scalar nonlinear function f. The information used to compute the k-th iterate is the values of $f, f', \ldots, f^{(d)}$ at all previous iterates. Proves that the maximal order of any such iterate is at most d+2. Settles a conjecture of Traub [64], at least for the case of iterations which use all previous information.

Busarova, T. N.,

"Best Quadrature Formulae for a Class of Differentiable and Periodic Functions," (in Russian),

Ukrainskij Mat. Zhurnal, 25, 1973, 291-301.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar periodic functions with bounded third derivative in L_∞. The information is the values of f or f and f'. Shows that the optimal points of information are equidistant and the coefficients of the optimal quadrature formulae are equal. The errors of the optimal algorithms are given.

Butcher, J. C.,

"On Runge-Kutta Processes of High Order,"

J. Austral. Math. Soc., 4, 1964, 179-194.

core, differential equations, Runge-Kutta methods, maximal order

Considers the solution of $y' = f(x,y)$, $y(x_0) = y_0$, by Runge-Kutta methods. The information is the values of f at n adaptively chosen points. Studies the maximal order p(n) of such methods. Finds the maximal order for n = 5 and improves the bounds for n = 6.

Butcher, J. C.,

"On the Attainable Order of Runge-Kutta Methods,"

Math. Comp. 19, 1965, 408-417.

core, differential equations, Runge-Kutta methods, maximal order

Continuation of Butcher [64]. Finds the maximal order for $n \le 9$ and the bound $p(n) \le n-2$ for $n \ge 10$.

Butcher, J. C.,

"An Order for Runge-Kutta Methods,"

SIAM J. Num. Anal., 12, 1975, 304-315.

core, differential equations, Runge-Kutta methods, maximal order

Continuation of Butcher [65]. Proves that there does not exist an explicit Runge-Kutta method which uses n evaluations of f and has order $p \ge u_k$ unless $n > p+k$, where $u_0 = 5$, $u_{n+1} = (4u_n + 2n + 3)/3$.

Casuli, V. and Trigiante, D.,

"Computational Complexity for a Class of Multipoint Iterative Procedures without or with Internal Memory,"

Calcolo, 14, 1977, 225-235.

core, nonlinear equations, scalar, multipoint iterations, complexity

Considers iterative solution of scalar nonlinear equations. As in Kung and Traub [74] ("Computational Complexity of One-Point and Multipoint Iteration") includes combinatory complexity in the complexity index.

Casuli, V. and Trigiante, D.,

"The Convergence Order for Iterative Multipoint Procedures,"

--*Calcolo*, 14, 1977, 25-44.

core, nonlinear equations, scalar, multipoint iterations, maximal order

Considers iterative solution of scalar nonlinear equations. Restrictive assumptions are made concerning the information and algorithms used. As in Kung and Traub [74] ("Optimal Order of One-Point and Multipoint Iterations") obtains maximal order for this class of iterations.

Chawla, M. M. and Kaul, V.,

"Optimal Rules for Numerical Integration Round the Unit Circle,"

BIT, 13, 1973, 145-152.

core, integration, scalar, optimal points of information, optimal algorithms

Considers integration and approximation of a linear functional for a class of scalar analytic functions defined on a circular annulus in a Hilbert space with a reproducing kernel. The information is the values of f. Optimal weights and points of information are derived in terms of the representers of the functionals. Optimal quadrature formulae for the unit circle and the interval [-1,1] are presented.

Chentsov, N. N.,

"On Quadrature Formulae for Functions of an Infinitely Large Number of Variables," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 1, 1961, 418-424.

English translation in U.S.S.R. Computational Math. and Math. Phys., 1, 1961, 455-464.

core, integration, abstract, lower bounds

Considers integration for a class of scalar functions of infinitely many variables. The information is the values of f. A sharp lower bound on the error of linear quadrature formulae for the class of Lipschitz functions is found.

Chernogorov, V. G.: See Bojanov, B. D.

Chernousko, F. L.,

"An Optimal Algorithm for Finding the Roots of an Approximately Computed Function," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 8, 1968, 705-724.

English translation in U.S.S.R. Computational Math. and Math. Phys., 8, 1968, 1-24.

core, nonlinear equations, scalar, optimal points of information, optimal error algorithms

Considers the search for a zero in the class of scalar functions f such that $m \le (f(x_1)-f(x_2))/(x_1-x_2) \le M$ for $x_1,x_2 \in [a,b]$ and $m > 0$. The information is the perturbed values of f. The optimal error algorithm is derived. Optimal points of information are obtained. Assuming that the cost of evaluating f to a certain accuracy is measured by a given cost function, the optimal error algorithm with fixed cost is discussed.

Chernousko, F. L.,

"Optimal Search for Extrema of Unimodal Functions," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 10, 1970, 922-933.

English translation in U.S.S.R. Computational Math. and Math. Phys., 10, 1970, 146-161.

core, extremum, scalar, optimal error algorithms

Considers the search for the extremum in the class of unimodal scalar functions satisfying a Lipschitz condition with a given constant. The information is the values of f at n points. The optimal error algorithms with respect to two different criteria are found. Results are related to those of Kiefer [53].

Chernousko, F. L.,

"Optimal Search for the Minimum of Convex Functions," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 10, 1970, 1355-1366.

English translation in U.S.S.R. Computational Math. and Math. Phys., 10, 1970, 20-34.

core, extremum, scalar, convex functions, optimal error algorithms

Considers the search for the minimum in the class of convex scalar functions. The information is the values of f at n points. The optimal or nearly optimal error algorithms are presented.

Chzhan, Guan-Tszynan,

"On the Minimum Number of Interpolation Points in the Numerical Integration of the Heat-Conduction Equation," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 2, 1962, 80-88.

English translation in U.S.R.R. Computational Math. and Math. Phys., 2, 1962, 78-87.

core, differential equations, n-widths

Considers the heat-conduction equation $u_t = u_{xx}$ where $u(x,0)$ belongs to the class H_p of functions f such that $f^{(2i)}(0) = f^{(2i)}(\pi) = 0$ for $2i < p$ and $f^{(p)} \in L_2$. The information is the values of $u(x,0)$. Studies the problem of the minimal number n of evaluations of $u(x,0)$ to approximate $u(x,t)$ to within ϵ by means of an algorithm whose range has dimension at most n. Derives an algorithm with $n = \Theta(\epsilon^{-1/p})$. Since n is the largest number such that $d_n(H_p) \le \epsilon$ where $d_n(H_p) = \Theta(\epsilon^{-1/p})$ is the Kolmogorov n-width, asymptotic optimality of the algorithm follows.

Cohen, A. I. and Varaiya, P.,
"Rate of Convergence and Optimality Conditions of Root Finding and Optimality Algorithms,"
Department of Electrical Engineering and Computer Science Report, University of California, Berkeley, 1970.

core, maximal order, one-point iteration, iterations with memory, scalar and multivariate nonlinear equations

Considers maximal order of iterations for solving scalar and multivariate nonlinear equations, $f = 0$. Information is evaluation of f and its derivatives at one or more points. Following S. Winograd, shows it may be possible to encode memory in an infinite precision number and therefore change maximal order of a class of algorithms. Adds condition to definition of order which ensures that encoding does not affect order. Finds maximal order of a class of algorithms.

Coman, Gh.,
"Monosplines and Optimal Quadrature Formulae in L_p,"
Rend. Mat., 5, 1972, 567-577.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar functions whose rth derivative is bounded in L_p by a constant. The information is the values of $f, f', \ldots, f^{(r-1)}$. Based on monosplines with least deviation, the weights and points of optimal linear quadrature formulae are derived.

Coman, Gh. and Micula, Gh.,

"Optimal Cubature Formulae,"

<u>Rend. Mat.</u>, 4, 1971, 303-311.

core, integration, multivariate, optimal points of information, optimal linear algorithms

Considers integration for a class of scalar functions of two variables with bounded derivatives in L_2. The information is the values of f and its derivatives. The weights and points of optimal linear quadrature formulae are derived.

Cooper, G. J. and Verner, J. H.,

"Some Explicit Runge-Kutta Methods of High Order,"

<u>SIAM J. Num. Anal.</u>, 9, 1972, 389-405.

core, differential equations, Runge-Kutta methods

Considers the solution of $y' = f(y)$, where y and f are vectors, by explicit Runge-Kutta methods. The information is the values of f. Studies Runge-Kutta methods of high order.

Cooper, L.: See Krolak, P.

Danilin, Yu. M.,

"On One Algorithm Efficiency Estimation of Absolute Minimum Finding," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 11, 1971, 1026-1030.

English translation: Danilin, Yu. M., "Estimation of the Efficiency of an Absolute-Minimum-Finding Algorithm," in U.S.S.R. Computational Math. and Math. Phys., 11, 1971, 261-267.

core, extremum, scalar, optimal error algorithms

Considers the search for the minimum in the class of scalar functions satisfying a Lipschitz condition with a given constant. The information is the values of f. Proves that Piavsky's algorithm (see Piavsky [72]) requires at most three times as many function evaluations as an optimal algorithm which solves the problem to within error ϵ.

Eckhardt, U.,

"Einige Eigenschaften Wilfscher Quadraturformeln,"

Numer. Math., 12, 1968, 1-7.

core, integration, scalar, optimal points of information, optimal algorithms

Considers integration for a class of scalar analytic functions on the unit disc in a Hilbert space. The information is the values of f. The existence and properties of optimal weights and points of quadrature formulae in the sense of Wilf [64] are presented.

Ehrmann, H.,

"Konstruktion und Durchführung von Iterationsverfahren Höherer Ordnung,"

Arch. Rational Mech. Anal., 4, 1959, 65-88.

core, polynomials, Newton iteration

In the complexity model of this paper, the author discusses what order iteration is best for computing a zero of a polynomial as a function of degree.

Eichhorn, B. H.,

"On Sequential Search,"

Selected Statistical Papers, 1, Math. Centrum, Amsterdam, 1968, 81-95.

core, extremum, nonlinear equations, scalar, optimal algorithms, average case analysis

Considers the search for the maximum or zero in the class of scalar unimodal functions or in the class of monotone nonincreasing functions with one zero, respectively. The information is the values of f at adaptively chosen points. Optimal algorithms for the worst and average cases are discussed.

Elhay, S.,

"Optimal Quadrature,"

Bull. Austral. Math. Soc., 1, 1969, 81-108.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for a class of smooth scalar functions for which $\sum_{j=0}^{r} a_j^2 \| f^{(j)} \|_2^2$ is bounded, a_j a real constant, $a_0 \neq 0$, $a_r \neq 0$. The information is the values of $f, f', \ldots, f^{(r-1)}$. Optimal quadrature formulae with optimal points of information are derived for $r = 1$ and 2. Some properties of optimal quadrature formulae are found for any r.

Emelyanov, K. V. and Ilin, A. M.,

"Number of Arithmetical Operations Necessary for the Approximate Solution of Fredholm Integral Equations of the Second Kind," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 7, 1967, 905-910.

English translation in U.S.S.R. Computational Math. and Math. Phys., 7, 1967, 259-266.

core, integral equations, lower and upper bounds

Considers the integral equation $y(P) = \int_D K(P,Q)y(Q)dQ + f(P)$ where $D \subset R^m$, for the class of functions K and f whose r-th derivatives are bounded. The information is the values of K and f. Shows that a lower bound on the error of any algorithm using n evaluations of K is at least $n^{-r/(2m)}$. The algorithm whose error achieves this bound is derived.

Feldstein, A.,

"Bounds on Order and Ostrowski Efficiency for *Interpolatory Iteration* Algorithms," UCRL-72238, Report Lawrence Livermore Laboratory, 1969.

core, nonlinear equation, scalar, interpolatory iterations, order, complexity index, parallel algorithm

Continues study of HIFs for the calculation of a zero of a nonlinear equation. Establishes bounds and limit theorems on the order of HIFs. Uses these results to establish bounds and limit theorems on a complexity index. Obtains such results for simple parallel iterations.

Feldstein, A. and Firestone, R. M.,

"Hermite Interpolatory Iteration Theory and Parallel Numerical Analysis," Division of *Applied Mathematics* Report, Brown University, 1967.

core, nonlinear equations, scalar, interpolatory iterations, order, complexity index, parallel algorithms

Studies Hermite interpolatory iteration functions (HIFs) for a zero of a nonlinear function f. The information is values of f and its derivatives at a number of points. Analyzes order of a HIF and studies a complexity index. Discusses application of HIF to parallel computation and compares complexity indices of parallel algorithms.

Feldstein, A. and Traub, J. F.,
"Order of Vector Recurrences with Applications to Nonlinear Iterations, Parallel Algorithms, and the Power Method,"
Department of Computer Science Report, Carnegie-Mellon University, 1974.

core, nonlinear equations, scalar, iterative complexity, iterative algorithms, parallel algorithms, order, iterations with memory, interpolatory iterations, composition

Same as Feldstein and Traub [77] with many examples of applications.

Feldstein, A. and Traub, J. F.,
"Asymptotic Behavior of Vector Recurrences with Applications,"
Math. Comp., 31, 1977, 180-192.

core, nonlinear equations, scalar, iterative complexity, iterative algorithms, parallel algorithms, order, iterations with memory, interpolatory iterations, composition

Proves under very weak assumptions that the root and quotient orders of the vector recurrence $\underline{y}_{n+1} = M\underline{y}_n + \underline{w}_{n+1}$ is the spectral radius of M. Continues work of Rice, Rice [71], on assigning a matrix representation to iterations for solving nonlinear equations. Applies this result to the analysis of parallel iteration algorithms and to the order and complexity of composite iterations. Shows that a composite iteration may have order less than, equal to, or greater than the products of the individual iterations.

Fine, T.,

"Optimum Search for the Location of the Maximum of a Unimodal Function," IEEE Trans. Information Theory, IT-12, 1966, 103-111.

core, extremum, scalar, optimal algorithms, average case analysis

Considers the search for the maximum in the class of unimodal functions assuming that the maximum points are uniformly distributed. The information is the values of f at adaptively chosen points. Optimal algorithms, i.e., algorithms which minimize the expected cost, are studied. See the referee's report by J. Kiefer in Math. Reviews, 34, No. 7260, where the model of the paper is discussed.

Firestone, R. M.: See Feldstein, A.

Forst, W.,

"Zur Optimalität interpolatorischer Quadraturformeln periodischer Funktionen," Numer. Math., 25, 1975, 15-21.

core, integration, scalar, optimal linear algorithms

Considers integration for the Favard class of scalar periodic functions whose rth derivatives satisfy a Lipschitz condition with unity. The information is the values of f at n equidistant points. Shows that the trapezoidal rule is the unique optimal quadrature formula and that its error is K_{r+1}/n^{r+1}, where K_{r+1} is the Favard constant.

Forst, W.,

"Optimale Hermite-Interpolation Differenzierbarer Periodischer Funktionen,"

J. Approximation Theory, 20, 1977, 333-347.

core, interpolation, scalar, optimal algorithms, splines

Considers interpolation for the Favard class of 2π-periodic scalar functions whose r-th derivative is bounded in L_∞ by unity. The information is the values of f and its derivatives. Optimal algorithms are derived in terms of periodic splines.

Gaffney, P. W.,

"Optimal Interpolation,"

D. Phil. Thesis, Oxford University, 1976.

core, interpolation, scalar, optimal error algorithms, splines

This is a Ph.D. thesis. Some of the results are published in Gaffney [77a, 77b] and Gaffney and Powell [76].

Gaffney, P. W.,

"The Range of Possible Values of f(x),"

Computer Science and Systems Division Report, AERE, Harwell, Oxfordshire, 1977.

core, interpolation, scalar, optimal error algorithms, perfect splines, lower bounds

Considers the interpolation problem for the class of scalar functions with bounded k-th derivative in L_∞. The information is the values of f at n points. The range of possible values of f(x) is bounded by interpolatory perfect splines of degree k. The computation of these splines at x is considered.

Gaffney, P. W.,

"To Compute the Optimal Interpolation Formula,"

Computer Science and Systems Division Report, AERE, Harwell, Oxfordshire, 1977.

core, interpolation, scalar, optimal error algorithms, splines

Considers the interpolation problem for the class of scalar functions with bounded k-th derivative in L_∞. The information is the values of f at n points. The optimal error algorithm is defined by an interpolatory spline Ω of degree k-1 with exactly n-k knots. The error is expressed by a perfect spline B of degree k with the same n-k knots as Ω. The computation of knots and coefficients of Ω and B is studied.

Gaffney, P. W. and Powell, M. J. D.,

"Optimal Interpolation,"

in Numerical Analysis, edited by G. A. Watson, Lecture Notes in Math., Vol. 506, Springer Verlag, 1976, 90-100.

core, interpolation, scalar, optimal error algorithms, splines

Considers interpolation for the class of scalar functions with bounded kth derivative in L_∞. The information is the values of f. The range of possible values f(x) for functions with the same information is derived in terms of perfect splines. The optimal error algorithm and its error are presented.

Gaisarian, S. S.,

"An Optimal Algorithm for the Approximate Computation of Quadratures," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 9, 1969, 1015-1023.

English translation in U.S.S.R. Computational Math. and Math. Phys., 9, 1969, 42-53.

core, integration, scalar, optimal points of information

Considers quadrature formulae of degree s. The information is the values of f. Minimizing the dominant error term, the optimal points of information are derived in terms of the sth derivative of f.

Gaisarian, S. S.,

"The Choice of Optimal Networks for the Numerical Solution of the Cauchy Problem for a Set of Ordinary Differential Equations,"

Zh. vychisl. Mat. mat. Fiz., 10, 1970, 465-474.

English tranlsation in U.S.S.R. Computational Math. and Math. Phys., 10, 1970, 253-267.

core, ordinary differential equations, optimal points of information

Considers the system of ordinary differential equation $y' = f(x,y)$, $y(x_0) = y_0$. The information is the values of f. The problem is solved by a one step method on the net $\{x_i\}$. The points $\{x_i\}$ which minimize the dominant error term are studied. Asymptotically optimal nets are found.

Gaisarian, S. S.: See also Tikhonov, A. N.

Gal, S.,

"Multidimensional Minimax Search for a Maximum,"

SIAM J. Appl. Math., 23, 1972, 513-526.

core, extremum, multivariate, optimal algorithms

Considers the search for the maximum in the class of linearly unimodal scalar functions of several variables. The information is the values of f at adaptively chosen points. Optimal algorithms are defined as algorithms which minimize the measure of the set of maxima of all functions satisfying the computed information. Proves that after n evaluations of f, the optimal error is at most roughly $(3/4)^n$. Also considers the same problem for the subclass of spherical symmetric functions.

Gal, S. and Micchelli, C. A.,

"Optimal Sequential and Non-Sequential Procedures for Evaluating a Functional," University of Wisconsin-Madison Report 1871, 1978. To appear in Applicable Analysis.

core, approximation of linear functionals and operators, optimal adaptive and non-adaptive information

Considers approximation of a linear functional (or operator) in a given class. The information is the values, possibly perturbed, of n linear functionals. Optimal deterministic, random and adaptive sequential information are studied. Shows that for many problems the optimal deterministic and adaptive sequential information yield the same error.

Ganshin, G. S.,

"Calculation of the Greatest Value of Function," (in Russian), Zh. vychisl. Mat. mat. Fiz., 16, 1976, 30-39.

English translation, Ganshin, G. S., "Function Maximization," U.S.S.R. Computational Math. and Math. Phys., 16, 1976, 26-36.

core, extremum, multivariate

Considers the search for the maximum in the class of scalar functions of several variables with a certain derivative bounded. The information is the values of f. The total number of evaluations of f needed to solve the problem to within ϵ is estimated.

Ganshin, G. S.,

"Optimal Algorithms of Calculation of the Function Highest Value," (in Russian),

Zh. vychisl. Mat. Mat. Fiz., 17, 1977, 562-572.

core, extremum, scalar, optimal points of information, optimal error algorithms

Considers the search for the maximum in three classes of scalar functions. The information is the values of f at n points. The optimal error algorithms are derived by finding the optimal net of n points of the given interval. The minimum value of n for which the error is at most ϵ is also derived.

Garey, M. R. and Johnson, D. S.,

"Approximation Algorithms for Combinatorial Problems: An Annotated Bibliography," in Algorithms and Complexity: New Directions and Recent Results, edited by J. F. Traub, Academic Press, New York, 1976, 41-52.

survey, combinatorial complexity

Gives annotated bibliography on papers with polynomial time approximation algorithms for combinatorial problems which have no known polynomial time algorithm.

Garey, M. R. and Johnson, D. S.,

Computers and Intractability,

W. H. Freeman, San Francisco, 1979.

survey, combinatorial complexity

A text on NP-complete problem. Contains material on approximate solution of hard problems. Good bibliography.

Gentleman, W. M.,

"Measures of Efficiency,"

unpublished letter to J. F. Traub, 1970.

core, iterative complexity

Proves that any efficiency index which is invariant under self-composition must be of a certain form. Although never published, the result has been widely quoted.

Germeier, Ju. B.,

Introduction to the Theory of Operations Research, (in Russian),

Nauka, Moscow, 1971.

mathematics and core, operations research, extremum, scalar, optimal algorithms

Considers among other problems the search for the maximum in the class of scalar functions satisfying a Lipschitz condition. The information is the values of f. An optimal algorithm is given.

Giršovič, Ju. M.: See Levin, M. I.

Golomb, M.,

"Interpolation Operators as Optimal Recovery Schemes for Classes of Analytic Functions,"

in Optimal Estimation in Approximation Theory, edited by C. A. Micchelli and T. J. Rivlin, Plenum Press, New York, 1977, 93-137.

core, approximation, optimal linear algorithms, splines, n-widths

Considers approximation for a class of complex-valued functions which have a reproducing kernel in a Hilbert space. The information is the values of f. The linear optimal error algorithm is shown to be an interpolatory spline. The optimal points of information and relation to n-widths are considered. For particular spaces of analytic functions, interpolatory splines and optimal errors are explicitly found.

Golomb, M. and Weinberger, H. F.,

"Optimal Approximation and Error Bounds,"

in <u>On Numerical Approximation</u>, edited by R. E. Langer, The University of Wisconsin Press, Madison, 1959, 117-190.

core, approximation of linear functionals, optimal error algorithms

This is one of the first papers dealing with approximation of a linear functional for a given class of elements. The information is the values of n functionals. Different conditions assuring the existence of optimal error algorithms with finite error are presented. The optimal error algorithms are extensively studied for a number of absorbing classes. The linear optimal error algorithms are derived for a Hilbert case. Although the word "spline" is not used, these algorithms are primarily based on splines. Many valuable examples illustrate the paper.

Grebennikov, A. I.,

"On Optimal Approximation of Nonlinear Operators," (in Russian),

<u>Zh. vychisl. Mat. mat. Fiz.</u>, 18, 1978, 762-768.

core, approximation of nonlinear operators, splines

Considers approximation of B(u) where B is a nonlinear operator from a Hilbert space H into a linear normed space and $\|Lu\| \leq R$ where L is a linear operator from H into a Hilbert space, R is a given constant. The information is the values of n linear functions $A_u = [L_1(y),\ldots,L_n(u)]$. The algorithm $\varphi(A_u) = B(u_f)$ where u_f is an interpolatory spline is introduced. Shown that φ is optimal within a factor of two. Conditions assuring optimality of φ are mentioned.

Grebennikov, A. I. and Morozov, V. A.,
"On Optimal Approximation of Operators," (in Russian),
Zh. vychisl. Mat. mat. Fiz., 17, 1977, 3-15.

core, approximation of linear operators, optimal linear algorithms

Considers primarily the approximation of a linear operator in a Hilbert space. The information is a linear operator. The existence of a linear optimal error algorithm is proven. Perturbed information is also studied. The paper is related to the concepts of central and spline algorithms.

Gross, O. and Johnson, S. M.,
"Sequential Minimax Search for a Zero of a Convex Function,"
MTAC (now Math. Comp.), 13, 1959, 44-51.

core, nonlinear equations, scalar, convex functions, optimal points of information, optimal algorithms

Considers the solution of nonlinear scalar equations for a class of convex continuous functions f such that $f(a) > 0$ and $f(b) < 0$. The information is the values of f at n adaptively chosen points. Studies optimal points of information, optimal algorithms and their errors.

Haber, S,

"The Error in Numerical Integration of Analytic Functions,"

Quart. Appl. Math., 29, 1971, 411-420.

core, integration, analytic functions, scalar, optimal linear algorithms, upper bounds

Considers integration for two classes of scalar analytic functions. The information is the values of f at n points. Shows upper bounds on the minimal error of quadrature formulae.

den Heijer, C.,

"On the Local Convergence of Newton's Method,"

Department of Numerical Mathematics Report, Mathematisch Centrum, Amsterdam, 1976.

core, nonlinear equations, abstract, Newton iteration, optimal convergence

Establishes the optimal radius of the ball of convergence for Newton iteration for the zero of a nonlinear operator. See Traub and Woźniakowski [77] "Convergence and Complexity of Newton Iteration for Operator Equations."

Herzberger, J.,

"Über Matrixdarstellungen für Iterationsverfahren bei Nichtlinearen Gleichurgen,"

Computing, 12, 1974, 215-222.

core, nonlinear equations, scalar, iterative algorithms, parallel algorithms, order, interpolatory iterations, composition

Shows that the order of convergence of certain iterations for the solution of nonlinear equations is the spectral radius of a certain matrix. See also Feldstein and Traub [77].

Hindmarsh, A. C.,

"Optimality in a Class of Rootfinding Algorithms,"

<u>SIAM J. Numer. Anal.</u>, 9, 1972, 205-214.

core, nonlinear equations, scalar, iterative complexity, iterative algorithms, order, interpolatory iterations

Studies Hermite interpolatory iteration functions (HIF's) for solving nonlinear equations. Shows how the order of composition of HIF's may be computed and that the order of a composite HIF is not the product of the orders. Studies the complexity of composite HIF's and discusses optimal complexity.

Hyafil, L.,

"Optimal Search for the Zero of the $(n-1)^{st}$ Derivative,"

IRIA/LABORIA Report No. 247, 1977.

core, nonlinear equations, scalar, optimal points of information, optimal error algorithm

Considers the search for a zero of the (n-1)-st derivative in the class of scalar functions f for which $f^{(n-1)}$ is continuous in [a,b], changes sign only once, and $f^{(n-1)}(a) < 0$, $f^{(n-1)}(b) > 0$. The information is the values of f at n points. Optimal error algorithms with optimally chosen points of information are derived for odd n and n = 2 or n = 4. The proofs are based on the zero finding problem for an asynchronous multiprocessor.

Ibragimov, I. I. and Aliev, R. M.,

"Best Quadrature Formulas for Certain Classes of Functions," (in Russian),

Dokl. Akad. Nauk SSSR, 162, 1965, 23-25.

English translation in Soviet Math. Dokl., 6, 1965, 621-623.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar functions with bounded r-th derivative in L_q for $q = 1,2$ or $+\infty$. The information is the values of $f, f', \ldots, f^{(r-2)}$ at m points. Optimal linear algorithms with optimal points of information and their errors are derived for even r.

Ilin, A. M.: See Emelyanov, K. V.

Ivanov, V. V.,

"On Optimal Algorithms for Function Minimization on Certain Classes of Functions," (in Russian),

Kibernetika 4, 1972, 81-94.

core, extremum, multivariate, optimal algorithms, error bounds

Considers the search for the minimum in the class of scalar functions of m variables with r-th derivative satisfying a Lipschitz condition with a given constant. The information is the values of f and its derivatives at n points. Shows that the error of an optimal algorithm is $\Theta(n^{-(r+1)/m})$. Also presents asymptotically optimal algorithms for the classes of analytic and entire functions.

Ivanov, V. V.,

"On Optimal Algorithms for the Calculation of Singular Integrals," (in Russian),

Dokl. Akad. Nauk SSSR, 204, 1972, 21-24.

English translation in Soviet Math. Dokl., 13, 1972, 576-580.

core, integration, scalar, optimal points of information, optimal algorithms

Considers a certain singular integration for a class of scalar functions. The information is the values of f. The optimal algorithms, their errors, and optimal points of information are given for several classes of functions.

Ivanov, V. V.,

"On Optimal in Accuracy Algorithms for Approximate Solution of Operator Equations of the First Kind," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 15, 1975, 3-12.

English translation, Ivanov, V. V., "Algorithms of Optimal Accuracy for the Approximate Solution of Operator Equations of the First Kind," in U.S.S.R. Computational Math. and Math. Phys., 15, 1975, 1-9.

core, operator equations, optimal error algorithms

Considers approximation of an element $R = O(I)$ for I belonging to a given set where O is an operator. The information is the value of $\Psi(I)$ for an operator Ψ. The optimal error algorithms are found. This idea is illustrated by the solution of $K\Phi = f$ where K is a linear compact operator for the class of f such that $f = KLu$, L is a given linear compact operator, $\|u\| \leq 1$. The information is $\Psi(K,f) = (K_\epsilon, f_\epsilon)$ where $\|K-K_\epsilon\| \leq \epsilon$, $\|f-f_\epsilon\| \leq \epsilon$. The asymptotically optimal error algorithms are derived.

Ivanov, V. V.,

"On Optimal Algorithms Approximating Functions for Some Classes," (in Russian),

in Theory of Approximation of Functions, Nauka, Moscow, 1977, 195-200.

core, approximation, optimal error algorithms

This is a paper presented at the international conference in Kaluga, U.S.S.R., in 1975. Surveys optimality results for the approximation problem in some classes of functions.

Jankowska, J.,

"Multivariate Secant Method,"

Ph.D. thesis, University of Warsaw, first part of thesis, 1975. To appear in SIAM J. Num. Anal.

core, nonlinear equations, multivariate, secant iteration, order of information

Considers the iterative solution of a system of nonlinear equations $f = 0$. The information is the values of f. Studies optimal properties of multivariate secant iteration. Shows how the position of points at which f is evaluated influences the order of information.

Jankowski, M. and Woźniakowski, H.,

"Computational Complexity in Numerical Mathematics," (in Polish),

Matematyka Stosowana, 5, 1975, 5-27.

core, algebraic and analytic complexity

Surveys recent problems in algebraic and analytic complexity for some numerical problems.

Jarratt, P.,

"Some Efficient Fourth Order Multipoint Methods for Solving Equations,"

BIT, 9, 1969, 119-124.

core, nonlinear equations, scalar, multipoint iterations, iterative complexity, order

Derives a fourth order method for solving scalar nonlinear equations that uses one function and two first derivative evaluations. Discusses the complexity index.

Jarratt, P.,

"A Review of Methods for Solving Nonlinear Algebraic Equations in One Variable,"

in Numerical Methods for Nonlinear Equations edited by P. Rabinowitz, Gordon and Breach, New York, 1970, 1-26.

core, nonlinear equations, scalar, one-point iterations, one-point iterations with memory, multipoint iterations, order

Surveys iterative algorithms for computing a zero of a scalar nonlinear function f. The information is the values of f or the values of f and f'. Also contains some new material on comparing the computational efficiencies of various iterations.

Jeeves, T. A.,

"Secant Modification of Newton's Method,"

Comm. ACM, 1, 8, 1958, 9-10.

core, nonlinear equation, scalar, Newton iteration, secant iteration

Compares complexity of secant and Newton iteration for solving a scalar nonlinear equation $f = 0$. First to observe that if the cost evaluating f' is greater than 43 times the cost of evaluation f, then the secant iteration has lower complexity than Newton's iteration.

Jetter, K.,

"Optimale Quadraturformeln mit semidefiniten Peano-Kernen,"

Numer. Math., 25, 1976, 239-249.

core, integration, scalar, optimal algorithms

Considers integration for a class of m-times differentiable scalar functions. The information is the values of f. Expresses the error of a quadrature formula as $cf^{(m)}(\xi)$ and studies the minimization of $|c|$.

Johnson, L. W. and Riess, R. D.,

"Minimal Quadratures for Functions of Low-Order Continuity,"

Math. Comp. 25, 1971, 831-835.

core, integration, scalar, optimal linear algorithms, upper bounds

Considers integration for a class of scalar functions of low-order continuity and for a class of scalar analytic functions on the unit disc. The information is the values of f at n points. Establishes the existence of quadrature formulae with minimal error. For the class of analytic functions shows that the error of these formulae is at most $O(n^{-1})$.

Johnson, S. M.,
"Best Exploration for Maximum is Fibonaccian,"
RAND Corp. Report P-856, 1956.

core, extremum, scalar, optimal points of information, optimal error algorithms

Considers the search for the maximum in the class of scalar unimodal functions. The information is the values of f at n adaptively chosen points. Optimality of Fibonaccian search is proven. See Kiefer [53].

Johnson, S. M.: See also Gross, O.

Judin, D. B. and Nemirovsky, A. S.,
"A Bound of Information Complexity for Mathematical Programming Problems," (in Russian),
Ekonomika i Matematicheski Metody, 12, 1976, 128-142.

core, extremum, multivariate, optimal points of information, optimal algorithms, error bounds

Considers mathematical programming problems in terms of minimization of scalar functions of several variables. The information is the values of f and its derivatives. Shows how many evaluations of f are necessary to find an ϵ-approximation for the class of functions whose (k-1)-st derivative satisfies a Lipschitz condition, and for a class of convex functions. Asymptotically optimal algorithms are given.

Judin, D. B. and Nemirovsky, A. S.,

"Information Complexity and Effective Methods for the Solution of Convex Extremal Problems," (in Russian),

Ekonomika i Mathematicheskie Metody, 12, 1976, 357-369.

core, extremum, multivariate, optimal points of information, optimal algorithms, error bounds

Continuation of Judin and Nemirovsky [76]. Studies practical aspects of asymptotically optimal algorithms. Shows that for a class of convex problems in a Hilbert space, $\Theta(\epsilon^{-2})$ function evaluations are necessary and sufficient to find an ϵ-approximation.

Judin, D. B. and Nemirovsky, A. S.,

"Information Complexity for Strict Convex Programming," (in Russian),

Ekonomika i Mathematicheskie Metody, 13, 1977, 550-559.

core, extremum, multivariate, optimal points of information, optimal algorithms, error bounds

Continuation of Judin and Nemirovsky [76]. Shows that for a class of strictly convex functions, $\Theta(\ln \epsilon^{-1})$ function evaluations are necessary and sufficient to find an ϵ-approximation.

Kacewicz, B.,
"Integrals with a Kernel in the Solution of Nonlinear Equations in N Dimensions," Computer Science Department Report, Carnegie-Mellon University, 1975. See also J.ACM, 26, 1979, 239-249.

core, nonlinear equations, abstract, multivariate, iterative information, order of information

Considers iterations for the solution of the nonlinear operator equation $f = 0$. Defines a maximal order iteration $I^g_{-1,s}$ which uses the information consisting of evaluations of the first s derivatives of f and a certain "integral with kernel g". In his complexity model proves that for the multivariate case with large N, $I^g_{-1,1}$ has lower complexity than $I^g_{-1,s}$, $s \geq 2$, and lower complexity than any interpolatory iteration.

Kacewicz, B.,
"An Integral-Interpolation Iterative Method for the Solution of Scalar Equations," Numer. Math., 26, 1976, 355-365.

core, nonlinear equations, scalar, iterative information, order of information

Introduces the idea of using integral information for the solution of the nonlinear scalar equation $f = 0$. Defines an iteration $I_{-1,s}$ which uses the information consisting of evaluations of the first s derivatives of f and a certain integral. Establishes the maximal order of $I_{-1,s}$.

Kacewicz, B.,

"The Use of Integrals in the Solution of Nonlinear Equations in N Dimensions," in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 127-141.

core, nonlinear equations, multivariate, iterative information, order of information

Introduces the idea of using integral information for the solution of the nonlinear N-dimensional equation $f = 0$. Defines a maximal order iteration $I_{-1,s}$ which uses the information consisting of evaluations of the first s derivatives of f and a certain integral. Proves in his complexity model that $I_{-1,1}$ has lower complexity than $I_{-1,s}$, $s \geq 2$. Proves that $I_{-1,1}$ has lower complexity than any interpolatory iteration using standard information, for large N.

Kacewicz, B. and Woźniakowski, H.,

"A Survey of Recent Problems and Results in Analytic Computational Complexity," Mathematical Foundations of Computer Science 77, Lecture Notes in Computer Science No. 53, edited by J. Gruska, Springer-Verlag, 1977, 93-107.

core, nonlinear equations, abstract, iterative complexity

Surveys recent research in iterative complexity. Good bibliography.

Karlin, S.,
"Best Quadrature Formulas and Interpolation by Splines Satisfying Boundary Conditions," and
"The Fundamental Theorem of Algebra for Monosplines Satisfying Certain Boundary Conditions and Applications to Optimal Quadrature Formulae,"
in Approximations with Special Emphasis on Spline Functions, edited by I. J. Schoenberg, Academic Press, New York, 1969, 447-466 and 467-484.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers, among other problems, integration for the class of scalar functions with bounded r-th derivative in L_2. The information is the values of f at n points and the values of its derivatives at the end points. Optimal algorithms in the sense of Sard with fixed and varying points of information, and their relation to monosplines are studied.

Karlin, S.,
"Best Quadrature Formulas and Splines,"
J. Approximation Theory, 4, 1971, 59-90.

core, integration, scalar, optimal algorithms

Considers integration for the class of scalar functions whose n-th derivative is bounded in L_2 by unity. The information is the value of f and its derivatives at the endpoints of an integration interval. Optimal algorithms in the sense of Sard are studied. Shows the correspondence to splines satisfying boundary conditions.

Karp, R. M. and Miranker, W. L.,
"Parallel Minimax Search for a Maximum,"
J. Comb. Th., 4, 1968, 19-35.

core, extremum, scalar, optimal error algorithms, parallel algorithms

Considers the search for the maximum in a class of scalar unimodal functions. The information is the values of f. Parallel optimal search algorithms are studied. Results related to those of Kiefer [53] are established.

Kaul, V.: See Chawla, M. M.

Kautsky, J.,
"Optimal Quadrature Formulae and Minimal Monosplines in L_q,"
Journal of the Austral. Mat. Soc., 11, 1970, 48-56.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar functions whose r-th derivative is bounded in L_q. The information is the values of $f, f', \ldots, f^{(r-1)}$. The optimal quadrature formula with optimal points of information and its error are derived. The proof is based on monosplines of minimal L_q-norm.

Keast, P.,

"Optimal Parameters for Multidimensional Integration,"

SIAM J. Numer. Anal., 10, 1973, 831-838.

core, integration, multivariate, optimal algorithms

Considers integration for a class of scalar periodic functions of several variables with bounded Fourier coefficients. The information is the values of f. Optimal quadrature formulae with equidistant weights are considered. Related to Korobov [63].

Kiefer, J.,

"Sequential Minimax Search for a Maximum,"

Proc. Amer. Math. Soc., 4, 1953, 502-505.

core, extremum, scalar, optimal points of information, optimal error algorithms

Considers the search for the maximum in the class of scalar unimodal functions. The information is the values of f at n points. Proves the classic result that Fibonaccian search is the optimal error algorithm. The optimal error is $1/F_{n+1}$ where F_{n+1} is the $(n+1)$th Fibonacci number.

Kiefer has informed us that this work was done as an MIT Master's Thesis in 1948 but was only published later with the encouragement of J. Wolfowitz.

Kiefer, J.,

"Optimum Sequential Search and Approximation Methods under Minimum Regularity Assumptions,"

J. Soc. Indust. Appl. Math., 5, 1957, 105-136.

core, approximation of nonlinear functionals, optimal points of information, optimal error algorithms

Considers approximation of a nonlinear functional for a class of functions. The information is the values of f. Optimal error algorithms are studied for the search of the zero of a monotonic real function, the search of the point at which the maximum of a unimodal function is attained, and the integration of a real function which is nondecreasing or satisfies a Lipschitz condition with a given constant.

Knauff, W. and Kress, R.,

"Optimale Approximation Linearer Funktionale auf Periodischen Funktionen,"

Numer. Math., 22, 1974, 187-205.

core, approximation of linear functionals, optimal algorithms

Considers approximation of a linear bounded functional for a class of periodic scalar functions. The information is the values of f at n equidistant points. Linear optimal algorithms and their errors are derived.

Knauff, W. and Kress, R.,

"Optimale Approximation mit Nebenbedingungen an Lineare Funktionale auf Periodischen Funktionen,"

Numer. Math., 25, 1976, 149-159.

core, approximation of linear functionals, optimal algorithms

Considers approximation of a linear functional for a class of scalar periodic complex functions. The information is the values of f at n equidistant points. Linear optimal algorithms which satisfy given linear constraints are derived.

Kornejčuk, N. P.,

"Best Cubature Formulas for Some Classes of Functions of Many Variables," (in Russian),

Matematicheskie Zametki 3, 1968, 565-576.

English translation in Math. Notes, 3, 1969, 360-367.

core, integration, multivariate, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar functions of several variables with bounded modulus of continuity. The information is the values of f. Optimal points of information and optimal weights for linear cubature formulae are found. The error for optimal points is derived.

Kornejčuk, N. P.,

"New Results on Extremal Problems of the Theory of Quadratures," (in Russian), appendix to the second edition of S. M. Nikolskij, Quadrature Formulae, Moscow, 1974, 136-223.

core, integration, scalar, optimal linear algorithms

Surveys the results obtained since the first edition of Nikolskij's book in 1958. Considers the integration problem for the class of scalar functions with bounded rth derivative in L_p. The information is the values of $f, f', \ldots, f^{(\zeta)}$. Shows that the problem of optimal points of information and optimal weights of linear quadrature formulae corresponds to the minimal norm spline with dominant term t^r in the space L_q, $1/p + 1/q = 1$. The solution is obtained for $\zeta = r-1$, and $\zeta = r-2$ for odd r.

Kornejčuk, N.P.,

Extremal Problems of Approximation Theory, (in Russian), Nauka, Moscow, 1976.

mathematics, approximation, error bounds, n-widths

Surveys sharp error bounds on the best approximation for several classes of functions (mostly periodic functions with bounded derivative or bounded modulus of continuity). The Kolmogorov and linear n-widths and extremal subspaces are considered. Good Russian bibliography.

Kornejčuk, N. P. and Lušpaj, N. E.,

"Best Quadrature Formulas for Classes of Differentiable Functions and Piecewise-Polynomial Approximation," (in Russian)

Izv. Akad. Nauk SSSR, Ser. Mat., 33, 1969, 1416-1437.

English tranlsation in Math. U.S.S.R.-Izvestija, 3, 1969, 1335-1355.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers the integration problem for the class of scalar functions with bounded rth derivative in L_p. The information is the values of $f, f', \ldots, f^{(\zeta)}$. Shows that the problem of optimal points of information and optimal weights of linear quadrature formulae corresponds to the minimal norm spline with leading term t^r in the space L_q, $1/p + 1/q = 1$. The solution is obtained for $\zeta = r-2$, and $\zeta = r-3$ for odd r.

Korobov, N. M.,

Number Theory Methods in Approximation Analysis, (in Russian), Fizmatgiz, Moscow, 1963.

core, integration, multivariate, interpolation, integral equations

Considers integration for the class of periodic scalar functions of several variables with bounded Fourier coefficients. The information is the values of f. Optimal points of information for quadrature formulae with equal weights are considered. Computation of nearly optimal points is studied using some results from number theory. The results are applied to the interpolation and integral equation problems.

Korotkov, V. B.,

"A Lower Error Bound on Cubature Formulas," (in Russian),

Sibirskii Matematicheskii Zhurnal, 17, 1977, 1188-1191.

core, integration, multivariate, lower bounds

Considers the integration problem for a class E of scalar functions of several variables. The information is the values of f. Shows that a lower bound on the error for this problem depends on the imbedding operator from E to the space L_1. Shows a connection between the integration and approximation problems.

Korytowski, A.: See Adamski, A.

Kress, R.: See Knauff, W.

Krolak, P. and Cooper, L.,

"An Extension of Fibonaccian Search to Several Variables,"

Comm. ACM, 6, 1963, 639-641.

core, extremum, multivariate

Considers the search for the maximum in a class of continuous unimodal scalar functions of several variables. The information is the values of f. Presents an algorithm which is a generalization of a one-dimensional Fibonaccian search algorithm. Partial optimality of this algorithm is given in Krolak, P., " Property of the Krolak-Cooper Extension of Fibonaccian Search," SIAM Rev. 8, 1966, 510-517 and Krolak, P., "Further Extensions of Fibonaccian Search to Nonlinear Programming Problems," SIAM J. Control., 6, 1968, 258-265.

Krylov, V. I.,

<u>Approximate Calculation of Integrals</u>,

Macmillan, New York, 1962, Chapter 8, 133-149.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar functions whose r-th derivative is bounded in L_q. The information is the values of f. Optimal weights and points of quadrature formulae are studied. The solution is derived for small r.

Kukarkin, A. B.: See Zhileikin, Ya. M.

Kung, H. T.,

"A Bound on the Multiplicative Efficiency of Iteration,"

<u>Proceedings of Fourth Annual ACM Symposium on Theory of Computing</u>, 1972, 102-107. Revised paper in <u>JCSS</u> 7, 1973, 334-342.

core, nonlinear equations, algebraic numbers, scalar, optimal iterations, multipoint iterations, iterative complexity, maximal order

Proves that the multiplicative efficiency index for an iteration approximating an algebraic number is bounded from above by unity. The proof uses degree growth arguments which permit the removal of restrictions in Paterson [72]. Proves that the maximal order of any sequence generated by an iteration with M multiplications or divisions is bounded by [illegible]^M.

Kung, H. T.,

"The Computational Complexity of Algebraic Numbers,"

Proceedings of Fifth Annual ACM Symposium on Theory of Computing, 1973, 152-159. Revised paper in SIAM J. Numer. Anal. 12, 1975, 89-96.

core, nonlinear equations, algebraic numbers, scalar, optimal iteration, multipoint iterations, iterative complexity

Defines two multiplicative efficiency indices E and $\bar{E}$. Kung [73] proves that both indices are bounded by unity for an iteration approximating an algebraic number α. Proves if $E = 1$ then α is rational, while if $\bar{E} = 1$ then α is rational or quadratic irrational.

Kung, H. T.,

"On Computing Reciprocals of Power Series,"

Numer. Math., 22, 1974, 341-348.

core, Newton iteration, formal power series, fast algorithms, algebraic complexity

Introduces idea of using Newton iteration on formal power series to compute reciprocal of a power series fast. Shows complexity of computing first N terms of reciprocal is no greater than multiplying two Nth degree polynomials.

Kung, H. T.,

"Complexity of Numerical Computation,"

in Proceedings International Computing Symposium 1975, edited by E. Gelenbe and D. Potier, North Holland, Amsterdam, 1975, 247-252.

core, survey, algebraic complexity, iterative complexity

Surveys recent research and problems in algebraic and iterative complexity. Good bibliography.

Kung, H. T.,

"Synchronized and Asynchronous Parallel Algorithms for Multiprocessors,"

in Algorithms and Complexity: New Directions and Recent Results, edited by J. F. Traub, Academic Press, New York, 1976, 153-200.

core, nonlinear equations, scalar, iterative algorithms, asynchronous, parallel algorithms, average case analysis

Classifies parallel algorithms for multiprocessors as synchronized or asynchronous algorithms. Identifies and discusses important concepts involved in the design and analysis of the two types of algorithms. Applies concepts to three examples: search for zeros, iterative algorithms for solving linear systems and nonlinear scalar equations, and adaptive asynchronous algorithms. Considers what is the optimal number of processes. Good bibliography.

Kung, H. T.,

"The Complexity of Obtaining Starting Points for Solving Operator Equations by Newton's Method,"

in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 35-57.

core, nonlinear equations, abstract, global information, iterative complexity, lower and upper bounds

Includes both the complexity of searching for a starting iterate and the complexity of iteration. Discusses when is the optimal time for changing from the search phase to the iterative phase. Gives new procedure for obtaining starting points for Newton iteration.

Kung, H. T. and Traub, J. F.,

"Computational Complexity of One-Point and Multipoint Iteration,"

in Complexity of Computation, edited by R. M. Karp, SIAM-AMS Proceedings, Vol. 7, Amer. Math. Soc., 1974, 149-160.

core, nonlinear equations, scalar, iterative information, one-point iterations, multipoint iterations, iterative complexity, lower bounds

Introduces the inclusion of combinatory complexity as well as information complexity in the complexity measure. Defines optimal complexity over a class of algorithms. Lower and upper bounds on optimal complexity are obtained for certain families of iterations.

Kung, H. T. and Traub, J. F.,
"Optimal Order of One-Point and Multipoint Iterations,"
J.ACM, 21, 1974, 643-651.

core, nonlinear equations, scalar, multipoint iterations, maximal order

Studies iterations for computing zeros of nonlinear scalar functions f. The information is the values of f and its derivatives. Constructs multipoint iteration of order 2^{n-1} which uses n evaluations. Conjectures that any multipoint iteration without memory which uses this information can be of order at most 2^{n-1}.

Kung, H. T. and Traub, J. F.,
"Optimal Order and Efficiency for Iterations with Two Evaluations,"
SIAM J. Numer. Anal., 13, 1976, 84-99.

core, nonlinear equations, scalar, iterative complexity, maximal order

Considers rational iterations without memory for solving the scalar nonlinear equation $f = 0$. The information used is two evaluations of f or its derivatives. Proves maximal order is 2 which settles, for $n = 2$, a conjecture of Kung and Traub that an iteration using n evaluations without memory is of order at most 2^{n-1}. Shows that any rational two-evaluation maximal order iteration without memory must use either two evaluations of f or one evaluation of f and one of f'. Determines minimal combinatory complexity for both these cases.

Kung, H. T. and Traub, J. F.,
"All Algebraic Functions can be Computed Fast,"
J.ACM, 25, 1978, 245-260.

core, formal power series, Newton iteration, Newton polygon, fast algorithms, algebraic complexity

Using the Newton polygon and Newton iteration the first N terms of the expansion of any algebraic function can be computed with complexity no greater than multiplying two Nth degree polynomials.

Kung, H. T.: See also Brent, R. P.

Kuzovkin, A. I. and Tikhomirov, V. M.,
"On the Number of Operations for Finding the Minimum of Convex Functions," (in Russian),
Ekonomika i Matematicheskie Metody, 3, 1967, 95-103.

core, extremum, multivariate, optimal points of information, optimal algorithms, error bounds

Considers the search for the minimum in a class of scalar convex functions of several variables. The information is the values of f. Optimal algorithms are studied. Shows that $\Theta(\ln \epsilon^{-1})$ function evaluations and arithmetic operations are necessary and sufficient to find an ϵ-approximation.

Larkin, F. M.,
"Optimal Approximation in Hilbert Spaces with Reproducing Kernel Functions,"
Math. Comp., 24, 1970, 911-921.

core, approximation of linear functionals, optimal algorithms

Considers approximation of a linear functional L for a class of scalar functions in a Hilbert space with a reproducing kernel. The information is the values of f. Optimal algorithms are derived in terms of the representers of information. Shows for which functions f, L(f) is exactly approximated by an optimal algorithm. Proves that Gaussian quadrature formula is nearly optimal for the integration problem in the class of analytic functions within the region $|x| < r$ for large r.

Lee, J. W.,
"Best Quadrature Formulas and Splines,"
J. Approximation Theory, 20, 1977, 348-384.

core, integration, scalar, optimal linear algorithms

Considers integration for a class of scalar functions. The information is the values of f and its derivatives. Optimal quadrature formulae in the sense of Sard are studied.

Levin, A. Ju.,

"On an Algorithm for the Minimization of Convex Functions," (in Russian),

Dokl. Akad. Nauk SSSR, 160, 1965, 1244-1247.

English translation in Soviet Math. Dokl., 6, 1965, 286-289.

core, extremum, multivariate, convex functions, upper bounds

Considers the search for the minimum in the class of scalar convex functions of several variables which satisfies a Lipschitz condition. The information is the values of f and its gradient. Presents an algorithm which finds an ϵ-approximation and requires $O(\ln \epsilon^{-1})$ evaluations of f and its gradient.

Levin, M. I. and Giršovič, Ju. M.,

"Extremal Problems for Cubature Formulas," (in Russian),

Dokl. Akad. Nauk SSSR, 236, 1977, 1303-1306.

English translation in Soviet Math. Dokl., 18, 1977, 1355-1358.

core, integration, multivariate, optimal points of information, optimal linear algorithms

Considers integration for some classes of functions of two variables with bounded derivatives in L_q. The information is the values of f and its derivatives. Optimal linear algorithms with fixed and optimal points of information and their errors are derived.

Levin, M. I., Giršovič, Ju. M. and Arro, V. K.,

"Best Quadrature Formulas on Sets of Functions," (in Russian),

Dokl. Akad. Nauk SSSR, 226, 1976, 51-54.

English translation in Soviet Math. Dokl., 17, 1976, 46-50.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for some classes of scalar functions with bounded r-th derivative in L_q. The information is the values of f, and its derivatives at the end points. Optimal linear algorithms with optimal points of information and their errors are presented.

Ligun, A. A.,

"Exact Inequalities for Splines and Best Quadrature Formulas for Certain Classes of Functions," (in Russian),

Matematicheskie Zametki 19, 1976, 913-926.

English translation in Math. Notes, 19, 1976, 533-541.

core, integration, scalar, optimal points of information, optimal linear algorithms, splines

Considers the integration problem for the class of periodic functions with bounded rth derivative. The information is the values of f. Shows that the rectangle quadrature formula is optimal and that equidistant points of information are optimal. The error for the optimal points is derived. Proofs are based on exact estimates of the kth derivative of periodic splines in Orlicz spaces with uniform norm.

Ligun, A. A.: See also Zaliznyak, N. F.

Lipow, P. R.,

"Spline Functions and Intermediate Best Quadrature Formulas,"

SIAM J. Numer. Anal., 10, 1973, 127-136.

core, integration, scalar, optimal algorithms, splines

Considers integration of a class of scalar functions. The information is the values of f and its derivatives. Optimal quadrature formulae in the sense of Sard are studied. Optimal formulae are based on cardinal Hermite splines.

Lipson, J.,

"Newton's Method: A Great Algebraic Algorithm,"

Proceedings 1976 Symposium on Symbolic and Algebraic Computation, edited by R. D. Jenks, Assoc. Comput. Mach., New York, NY, 1976, 260-270.

core, formal power series, Newton iteration, fast algorithms, algebraic complexity

Shows that for a "regular" problem Newton iteration may be used to compute the first N terms of the expansion of any algebraic function with complexity no greater than multiplying two Nth degree polynomials. See also Kung and Traub [78], "All Algebraic Functions Can be Computed Fast."

Loeb, H. and Werner, H.,

"Optimal Numerical Quadrature in H_p Space,"

<u>Math. Z.</u> 138, 1974, 111-117.

core, integration, analytic functions, scalar, optimal linear algorithms, upper bounds

Considers integration for the class H_p of scalar analytic functions on the unit disc. The information is the values of f and its derivatives at n points. Proves that the upper bound on the minimal error of quadrature formulae is roughly exp(-cn) where $c > 0$.

Loeb, H. L.: See also Barrar, R. B.

Lušpaj, N. E.,

"Best Quadrature Formulae for Some Classes of Functions," (in Russian),

<u>Proceedings of the International Conference of Young Research Mathematicians</u>, Charkov, 1966, 58-62.

See Lušpaj [69].

Lušpaj, N. E.,

"Best Quadrature Formulas on Classes of Differentiable Periodic Functions," (in Russian),

Matematicheskie Zametki, 6, 1969, 475-482.

English translation in Math. Notes, 6, 1969, 740-744.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of 2π periodic functions with bounded rth derivative in L_q. The information is the values of $f, f', \ldots, f^{(\zeta)}$. The optimal points and weights of linear quadrature formulae are found for $\zeta = r-1$, $r = 1,2,\ldots$. The error of this linear optimal algorithm is also derived.

Lušpaj, N. E.,

"Best Quadrature Formula on the Class $W_*^r L_2$ of Periodic Functions," (in Russian),

Matematicheskie Zametki, 16, 1974, 193-204.

English tranlsation in Math. Notes, 16, 1974, 701-708.

core, integration, scalar, optimal linear algorithms, error bounds

Considers the integration problem for the class of periodic functions whose r-th derivative is bounded by unity in L_2. The information is the values of $f, f', \ldots, f^{(\zeta)}$ at n points. The linear optimal quadrature formula is found for $\zeta = r-2$ and $r-3$. The error bounds are derived.

Lušpaj, N. Ė.: See also Kornejčuk, N. P.

Majstrovskij, G. D.,

"On the Optimality of Newton's Method," (in Russian),

Dokl. Akad. Nauk SSSR, 204, 1972, 1313-1315.

English translation in Soviet Math. Dokl., 13, 1972, 838-840.

core, nonlinear equations, multivariate, Newton iteration

Considers the solution of nonlinear equations $f = 0$ for the class of functions f from R^n into R^n with a bounded Lipschitz constant. The information is the values of f and f'. Suppose that x_0 is a sufficiently close approximation to a solution. Then it is shown that any algorithm using n values of f and f' at any points has error essentially not less than the error of Newton's method after n steps.

Mangasarian, O. L. and Schumaker, L. L.,

"Best Summation Formulae and Discrete Splines,"

SIAM J. Numer. Anal., 10, 1973, 448-459.

core, approximation of linear functionals, optimal algorithms, splines

Considers approximation of a linear functional defined on a finite dimensional function space. The information is the values of f. Optimal algorithms in the sense of Sard are studied. The problem is reduced to a solvable linear or quadratic programming problem. Shows discrete monosplines are related to optimal algorithms.

Mansfield, L. E.,

"On the Optimal Approximation of Linear Functionals in Spaces of Bivariate Functions,"

SIAM J. Numer. Anal., 8, 1971, 115-126.

core, approximation of linear functionals, optimal algorithms

Considers approximation of a linear functional for a class of bivariate functions in a Hilbert space. The information is the values of n linear functionals. Optimal algorithms are derived in terms of the representers of the appropriate functionals. Application to integration is considered.

Mansfield, L. E.,

"Optimal Approximation and Error Bounds in Spaces of Bivariate Functions,"

J. Approximation Theory, 5, 1972, 77-96.

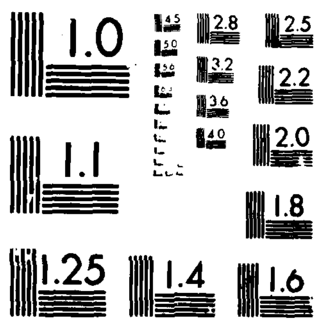
core, approximation of linear functionals, optimal algorithms

Considers approximation of a linear functional for a class of scalar functions of two variables in a Hilbert space with a reproducing kernel. The information is the values of n linear functionals. Optimal algorithms and error bounds are found in terms of the representers of the functionals. Application to the integration problem is considered. Related to bivariate splines.

AD-A081 439 CARNEGIE-MELLON UNIV PITTSBURGH PA DEPT OF COMPUTER --ETC F/G 12/1
ANNOTATED BIBLIOGRAPHY AND BRIEF HISTORY OF OPTIMAL ALGORITHMS --ETC(U)
AUG 79 J F TRAUB, H WOZNIAKOWSKI N00014-76-C-0370
UNCLASSIFIED CMU-CS-79-140 NL

2 OF 2
AD
A081439

END
DATE
FILMED
4-80
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Marchuk, A. G. and Osipenko, K. Yu.,

"Best Approximation of Functions Specified with an Error at a Finite Number of Points," (in Russian),

Matematicheskie Zametki, 17, 1975, 359-368.

English translation in Math. Notes, 17, 1975, 207-212.

core, approximation of linear functionals, optimal linear algorithms

Considers the approximation of a linear functional for a given class of problems. The information is the perturbed values of n linear functionals. A linear optimal error algorithm is constructed. This is a generalization of Smolyak's lemma.

Maung Čžo Njun and Sharygin, I. F.,

"Optimal Cubature Formulae on the Classes $D_2^{1,c}$ and D_s^{1,L_1}," (in Russian), in Problems of Numerical and Applied Mathematics, Tashkent, 1975, 5, 22-27.

core, integration, multivariate, optimal linear algorithms

Considers integration for two classes of scalar functions of two and s variables, respectively, with bounded first derivative. The information is the values of f. Presents asymptotically optimal linear algorithms.

Meinguet, J.,

"Optimal Approximation and Error Bounds in Seminormed Spaces,"

Num. Math. 10, 1967, 370-388.

core, approximation of linear functionals

Considers approximation of a linear functional for a given class. The information is the values of n linear functionals. The range of possible values of the functional for elements satisfying the information is derived.

Melkman, A. A.,

"n-Widths and Optimal Interpolation of Time- and Band-Limited Functions," in Optimal Estimation in Approximation Theory, edited by C. A. Micchelli and T. J. Rivlin, Plenum Press, New York, 1977, 55-68.

core, interpolation, optimal points of information, n-widths

Considers the optimal recovery of functions from a subset of the Paley Wiener class of entire functions. The information is the values of f. The linear optimal error algorithm is derived. The optimal points of information are studied. The error for optimal points is the n-width of the subset. The n-widths are found for a more general set of time- and band-limited functions.

Melkman, A. A. and Micchelli, C. A.,

"Optimal Estimation of Linear Operators in Hilbert Spaces from Inaccurate Data,"

Universität Bonn Report, 1977. See also <u>SIAM J. Num. Anal.</u>, 16, 1979, 87-105.

core, approximation of linear operators, optimal linear algorithms

Considers approximation of a linear operator for a class in a Hilbert space. The information is the value of a perturbed information operator. Linear optimal error algorithms are derived.

Meyers, L. F. and Sard, A.,

"Best Approximate Integration Formulas,"

<u>J. of Math. and Phys.</u>, 28, 1950, 118-123.

core, integration, scalar, optimal algorithms

Considers integration for a class of scalar m-times differentiable functions. The information is the values of f at n points. Optimal quadrature formulae in the sense of Sard are derived for small m and some n.

Meyers, L. F. and Sard, A.,

"Best Interpolation Formulas,"

<u>J. of Math. and Phys.</u>, 29, 1950, 198-206.

core, interpolation, scalar, optimal algorithms

Considers interpolation for a class of scalar m-times differentiable functions. The information is the values of f at n points. Optimal algorithms in the sense of Sard are derived for small n and m.

Micchelli, C. A.,

"Optimal Estimation of Linear Functionals,"

IBM Research Report 5729, 1975.

core, approximation of linear functionals and linear operators, optimal linear algorithms

Considers the approximation of a linear functional or operator for a balanced convex set. The information is the value of a perturbed linear operator. The existence of linear optimal error algorithms is studied.

Micchelli, C. A.,

"On an Optimal Method for the Numerical Differentiation of Smooth Functions,"

J. Approximation Theory, 18, 1976, 189-204.

core, differentiation, optimal linear algorithms

Considers the approximation of $f'(0)$ for the class of functions for which $\|Lf\| \leq \gamma$ where L is a polynomial differential operator and γ is a constant. The information is the values of a perturbed f at infinitely many points. A linear optimal error algorithm is derived.

Micchelli, C. A. and Miranker, W. L.,

"High Order Search Methods for Finding Roots,"

J.ACM, 22, 1975, 51-60.

core, nonlinear equations, scalar, globally convergent iterations, order

Defines and analyzes higher order globally convergent search methods for solving the nonlinear scalar equation $f = 0$. The information used are evaluations of f and global bounds on certain derivatives. The order of convergence of these methods is obtained.

Micchelli, C. A. and Pinkus, A.,

"On a Best Estimator for the Class M^r Using Only Function Values,"

Indiana University Mathematics Journal, 26, 1977, 751-759.

core, approximation, optimal points of information, optimal linear algorithms, splines, n-widths

Considers approximation for the class of functions $f(x) = P(x) + \frac{1}{(r-1)!}\int_0^1 (x-t)_+^{r-1}\, d\lambda_f(t)$ where P is a polynomial of degree $\le r$ and the total variation $\|\lambda_f\| \le 1$. The information is the values of f. The optimal error algorithm in L_1 is shown to be a linear interpolatory spline. The error for the optimal points of information is the Kolmogorov n-width of the class.

Micchelli, C. A. and Rivlin, T. J.,

"A Survey of Optimal Recovery,"

in Optimal Estimation in Approximation Theory, edited by C. A. Micchelli and T. J. Rivlin, Plenum Press, New York, 1977, 1-54.

core, approximation of linear operators, optimal error algorithms, optimal linear algorithms

Considers the optimal approximation of a linear operator U for a balanced convex class K. The information is the value of a perturbed linear operator I. Optimal error algorithms are studied. Assuming that U is a functional, the existence and properties of the linear optimal error algorithms are extensively studied. (It is not assumed that I is finite dimensional.) The ideas are illustrated by many valuable examples. In some cases, splines are shown to be a basic tool for optimal recovery.

Micchelli, C. A. and Rivlin, T. J.,

Optimal Estimation in Approximation Theory,

Plenum Press, New York, 1977.

core, approximation

This is the proceedings of the International Symposium on Optimal Estimation in Approximation Theory in 1976. Contains many papers of interest. See entries under Micchelli and Rivlin [77], Melkman [77], de Boor [77], and Golomb [77].

Micchelli, C. A. and Rivlin, T. J.,

"Optimal Recovery of Best Approximations,"

IBM T. J. Watson Research Center Report 7071, 1978. To appear in Resultate der Mathematik.

core, approximation, optimal error algorithms

Considers approximation of the best uniform polynomial approximation of a function from the class of continuous functions bounded in the sup norm by unity. The information is the perturbed values of f. The optimal error algorithm and its error are derived.

Micchelli, C. A., Rivlin, T. J. and Winograd, S.,

"The Optimal Recovery of Smooth Functions,"

Numer. Math., 26, 1976, 191-200.

core, approximation, optimal error algorithms, splines

Considers approximation for the class of scalar functions with bounded kth derivative in L_∞. The information is the values of f and its derivatives. The linear optimal error algorithm is shown to be essentially an interpolatory spline. Estimates of the optimal error are given.

Micchelli, C. A.: See also Gal, S.; Melkman, A. A.

Micula, Gh.: See Coman, Gh.

Miranker, W. L.,

"A Survey of Parallelism in Numerical Analysis,"

SIAM Review, 12, 1971, 524-547.

core, survey, nonlinear equations and extremum, scalar, parallel algorithms

Surveys parallel algorithms for numerical problems. Among the areas covered are the solution of nonlinear equations and optimization. Good bibliography.

Miranker, W. L.: See also Karp, R.; Micchelli, C. A.

Mitkowski, W.: See Adamski, A.

Mockus, I. B.,

"Bayesian Methods for Extremum Search," (in Russian),

Avtomat. i Vyčisl. Techn., 3, 1972, 53-62.

core, extremum, scalar, optimal Bayesian algorithms

Considers the search for the minimum in a class of scalar functions. Optimal Bayesian algorithms are presented.

Morozov, V. A.: See Grebennikov, A. I.

Motornyj, V. P.

"On the Best Quadrature Fromula of the Form $\sum_{k=1}^{n} p_k f(x_k)$ for Some Classes of Periodic Differentiable Functions," (in Russian),

Dokl. Akad. Nauk SSSR, 211, 1973, 1060-1062.

English translation in Soviet Math. Dokl., 14, 1973, 1180-1183.

core, integration, scalar, optimal points of information, optimal linear algorithms

This is an announcement of the results proven in Motornyj [74].

Motornyj, V. P.

"On the Best Quadrature Formulae of the Form $\sum_{k=1}^{n} p_k f(x_k)$ for Some Classes of Periodic Differentiable Functions," (in Russian),

Dokl. Akad. Nauk SSSR, Ser. Math., 38, 1974, 583-614.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of periodic scalar function whose rth derivative has bounded modulus of continuity. The information is the values of f. Optimal points of information and optimal quadrature formulae are studied. Optimality of rectangle quadrature with equidistant points is shown for three classes of functions.

Motornyj, V. P.,

"Some Extremal Problems of Theory of Quadrature and Approximation of Functions," (in Russian),

Matematicheskie Zametki, 19, 1976, 299-311.

English translation in Math. Notes, 19, 1976, 176-183.

core, integration, approximation, scalar, periodic functions

This is the abstract of a habilitation thesis. Surveys results on quadrature and approximation for several classes of periodic differentiable functions. Good Russian bibliography.

Munro, I.: See Borodin, A.

Nemirovsky, A. S.: See Judin, D. B.

Newman, D. J.,

"Location of the Maximum on Unimodal Surfaces,"

J.ACM, 12, 1965, 395-398.

core, extremum, multivariate, asymptotically optimal algorithms

Considers the search for the maximum on the lattice points in the class of scalar unimodal functions of k variables. The information is the values of f. Proves that asymptotically log n function evaluations are enough to solve the problem on the lattice with $(n+1)^k$ points.

Nielson, G. M.,
"Bivariate Spline Functions and the Approximation of Linear Functionals,"
Numer. Math., 21, 1973, 138-160.

core, approximation of linear functionals, splines

Considers approximation of a linear functional for a class of scalar functions of two variables in a Hilbert space. The information is the values of n linear functionals. Linear optimal algorithms are shown to be based on bivariate interpolatory splines.

Nikolskij, S. M.,
"On the Problem of Approximation Estimate by Quadrature Formulae," (in Russian), Uspekhi Mat. Nauk, 5, 1950, 165-177.

core, integration, scalar, optimal points of information, optimal linear algorithms

This classic paper considers integration for the class of scalar functions whose r-th derivative is bounded in L_p by a constant. The information is the values of $f, f', \ldots, f^{(\zeta)}$ at n points. Optimal linear quadrature formula are defined. The points and weights of an optimal quadrature formula are found for $r = 1,2$, and for arbitrary r with $\zeta = r-2$.

Nikolskij, S. M.,

Quadrature Formulae, (in Russian),

Nauka, Moscow, first edition 1958, second edition 1974.

English translation Delhi, Hindustan Pub. Corp. (India), 1964.

core, integration, scalar, optimal points of information, optimal linear algorithms

This classic book introduces optimal linear quadrature formulae for a class of functions. The information is the values of $f, f', \ldots, f^{(\zeta)}$. The optimal points of information are found for the class of functions with bounded rth derivative in L_∞ for small r or $\zeta = r-2$.

Nilson, E. N.: See Ahlberg, J. H.

Novikov, V. A.: See Aphanasjev, A. Yu.

Ortega, J.M., and Rheinboldt, W. C.,

Iterative Solutions of Nonlinear Equations in Several Variables,

Academic Press, New York, 1970.

mathematics, nonlinear equations, multivariate, order

This monograph surveys the basic theoretical results about nonlinear equations in n dimensions. Analyzes many iterations. Good bibliography.

Osipenko, K. Yu.,

"Optimal Interpolation of Analytic Functions," (in Russian),

Matematicheskie Zametki, 12, 1972, 465-476.

English translation in Math. Notes, 12, 1972, 712-719.

core, interpolation, analytic functions, scalar, optimal points of information, optimal linear algorithms

Considers the interpolation problem for the class of real scalar functions on [a,b] which can be analytically extended to a region G and whose extension is bounded by a constant. The information is the values of f at n points. Using Smolyak's lemma the linear optimal error algorithm and its error are derived. Optimal points of information are considered. If G is the unit disk, the optimal points are determined by elliptic functions. If G is the ellipsoid with foci $\pm$ 1 and che sum of semi-axes equal to c, the optimal points are the zeros of the Chebyshev polynomial and the error is roughly c^{-n}.

Osipenko, K. Yu.,

"Best Approximation of Analytic Functions from Information about their Values at a Finite Number of Points," (in Russian),

Matematicheskie Zametki, 19, 1976, 29-40.

English translation in Math. Notes, 19, 1976, 17-23.

core, approximation of linear functionals, analytic functions, scalar, optimal linear algorithms

Generalizes Smolyak's lemma to the complex case. A linear optimal error algorithm for the approximation of a linear complex functional is proven. The approximation of analytic scalar functions is considered. Results are related to Osipenko [72].

Osipenko, K. Yu.: See also Marchuk, A. G.

Ostrowski, A.,

Solution of Equations in Euclidean and Banach Spaces, Third Edition, Academic Press, New York, 1973.

mathematics, nonlinear equations, scalar, multivariate, abstract, iterative algorithms, one-point iterations, iterations with memory, order, Newton iteration, secant iteration

A monograph on the numerical solution of nonlinear equations. Contains much material on convergence and order of iterations. Surveys theory of solving equations numerically.

Pallashke, D.,

"Optimale Differentiations - und Integrationsformeln in $C_0[a,b]$,"

Numer. Math. 16, 1976, 201-210.

core, differentiation, integration, scalar, optimal algorithms

Considers differentiation and integration for a class of scalar continuous functions. The information is the values of f. Optimal algorithms are derived.

Parker, D. S. Jr.,

"Studies in Conjugation: Huffman Tree Construction, Nonlinear Recurrences, and Permutation Networks,"

Department of Computer Science Report UIUCDCS-R-78-930, Ph.D. Thesis, University of Illinois, 1978.

core, composition, formal power series, fast algorithms, parallel algorithms

Considers the evaluation of nonlinear recurrences on a parallel machine. Studies when there exists a transformation which makes this problem easy and whether this transformation process can be automated. Good bibliography. See also Brent and Traub [78].

Paszkowski, S.,

"Optimum Choice of Initial Approximations in Interpolation Methods of Solving Equations,"

Zastosowania Matematyki, 12, 1971, 201-216.

core, nonlinear equations, scalar, interpolatory iterations, optimal initial approximations

Considers the iterative solution of nonlinear scalar equation $f = 0$. The information is the values of f. Studies the optimal choice of initial approximations for interpolatory iterations. Shows that this problem is equivalent to minimization of the sup norm of the function $\prod_{i=0}^{n} |x-a_i|^{b_i}$ with respect to a_i, for $x \in [-1,1]$ and fixed positive b_i.

Paterson, M. S.,

"Efficient Iterations for Algebraic Numbers,"

in Complexity of Computer Computations, edited by R. E. Miller and J. W. Thatcher, Plenum Press, New York, 1972, 41-52.

core, nonlinear equations, algebraic numbers, scalar, optimal iteration, one-point iterations, iterative complexity

Introduces question of optimal iteration for approximation of algebraic numbers. Proves that the multiplicative efficiency index (which is the reciprocal of the complexity index) must be bounded from above by unity. The proof uses results from number theory. Presents several conjectures on optimal efficiency.

Paulik, A.,

"Zur Existenz Optimaler Quadraturformeln mit freien Knoten bei Integration analytischer Funktionen,"

Numer. Math., 27, 1977, 395-405.

core, integration, scalar, optimal linear algorithms

Considers integration for a class of scalar analytic functions on a circle. The information is the values of f. For fixed points of information the optimal quadrature and its error are derived. The existence of optimal points is established.

Piavsky, S. A.,

"One Algorithm for the Searching of Global Extremum of Function," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 1972, 888-896.

English translation in U.S.S.R. Computational Math. and Math. Phys., 1972, 57-67.

core, extremum, multivariate

Considers an algorithm finding the minimum for the class of functions $f : X \to R$, where X is a compact set, such that there exists $g : X \times X \to R$ and $f(x) \geq g(x,y)$, $\forall x, y \in X$. (If f satisfies a Lipschitz condition $|f(x)-f(y)| \leq L\|x-y\|$, set $g(x,y) = f(y) - M\|x-y\|$ for $M \geq L$.) The information on f is the function g. The algorithm requires finding a global minimum of $\max_{0 \leq j \leq k} g(x,y_j)$ for different $y_1,\ldots,y_k$. The efficiency analysis and optimality within a factor of 3 of this algorithm is proved by Danilin [71] for the scalar case.

Pinkus, A.,

"Asymptotic Minimum Norm Quadrature Formulae,"

Numer. Math., 24, 1975, 163-175.

core, integration, scalar, optimal linear algorithms

Considers integration for the class of scalar analytic functions whose norm is bounded by unity on a complex domain B. The information is the values of f and its derivatives. The optimal weights and points of linear quadrature formulae are studied. Shows that if B grows to the whole complex space, then the optimal weights and points converge to the weights and points of the Gaussian quadrature.

Pinkus, A.: See also Micchelli, C. A.

Pleshakov, G. N.,
"On Efficiency of the Multidimensional Interpolation Iterations," (in Russian), Zh. vychisl. Mat. mat. Fiz., 17, 1977, 1153-1160.

core, nonlinear equations, multivariate interpolatory iterations

Considers the solution of n nonlinear equations in n unknowns. The information is the values of f. Inverse interpolatory iterations with memory are studied. A necessary assumption on a suitable position of iteration points seems to be missing. The Ostrowski efficiency index is discussed.

Powell, M. J. D.: See Gaffney, P. W.

Reinsch, Ch.,
"Two Extensions of the Sard-Schoenberg Theory of Best Approximation," SIAM J. Numer. Anal., 11, 1974, 45-51.

core, approximation of linear functionals, optimal algorithms, splines

Considers approximation of a linear functional for a class of functions. The information is the values of f and its derivatives. Optimal algorithms based on natural or periodic splines are derived. Extensions are given to a Hilbert setting and to the problem of smoothing.

Rheinboldt, W. C.: See Ortega, J. M.

Rice, J. R.,

The Approximation of Functions, Vol. II, Chapter 10,

Addison Wesley, Reading, Mass., 1969.

core and mathematics, approximation of linear functionals, optimal error algorithms

Considers among other problems, approximation of a linear functional on the ball in a Hilbert space. The information is the values of n linear functionals. Optimal error algorithms are constructed using splines.

Rice, J. R.,

"Matrix Representations of Nonlinear Equation Iterations--Application to Parallel Computation,"

Math. Comp., 25, 1971, 639-647.

core, nonlinear equations, scalar, iterative algorithms, parallel algorithms, order, interpolatory iterations, composition

Introduces idea of assigning a matrix representation to iterations for solving nonlinear equations. Applies his results to the analysis of iterations useful in parallel computation. See also Feldstein and Traub [77].

Rice, J. R.,

"On the Computational Complexity of Approximation Operators,"

in Approximation Theory, edited by G. G. Lorentz, Academic Press, New York, 1973, 449-456.

See Rice [76].

Rice, J. R.,

"On the Computational Complexity of Approximation Operators II,"

in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 191-204.

core, approximation, optimal points of information, optimal algorithms

Continuation of Rice [73]. Considers approximation of scalar p-times differentiable or analytic functions. The information is the values of f. Assumes that the only operations counted are evaluations of f. Studies how many function evaluations are asymptotically necessary to produce an estimate with error proportional to the error of the best polynomial approximation for different classes of functions.

Richter, N.,

"Properties of Minimal Integration Rules,"

SIAM J. Numer. Anal., 7, 1970, 67-79.

core, integration, analytic functions, scalar, optimal algorithms

Considers integration for a class of scalar analytic functions in a Hilbert space. The information is the values of f. The existence of optimal quadrature formulae is proved. Properties of weights and points of an optimal quadrature are studied. See also Richter-Dyn, N., "Properties of Minimal Integration Rules, II," SIAM J. Numer. Anal., 8, 1971, 497-508.

Richter-Dyn, N.,

"Minimal Interpolation and Approximation in Hilbert Spaces,"

<u>SIAM J. Numer. Anal.</u>, 8, 1971, 583-597.

core, approximation of linear functionals, optimal linear algorithms

Considers approximation of a linear functional for a class of scalar functions in a Hilbert space with a reproducing kernel. The information is the values of f. Optimal linear algorithms are studied. Application to integration is considered.

Riess, R. D.: See Johnson, L. W.

Rissanen, J.,

"Maximum Power Feedback Law,"

<u>Int. J. Control</u>, 14, 1971, 233-240.

core, nonlinear equations, scalar, iterative information, control theory, maximal order

Considers the iterative solution of certain scalar nonlinear equations which occur in control theory. Obtains an iterative algorithm by a process of linearization. Shows this algorithm has order about 1.55 if certain initial conditions hold. Proves that this algorithm has "maximal power" among all "smooth" algorithms using the same information.

Rissanen, J.,

"On Optimum Root-Finding Algorithms,"

J. Math. Anal. Applic., 36, 1971, 220-225.

core, nonlinear equations, scalar, maximal order

Considers maximal order of algorithms for solving a scalar nonlinear equation. Proves that the secant method has maximal order among all algorithms using the same information as the secant method.

Ritter, K.,

"Two-Dimensional Spline Functions and Best Approximations of Linear Functionals,"

J. Approximation Theory, 3, 1970, 352-368.

core, approximation of linear functionals, optimal algorithms, splines

Considers approximation of a linear functional for a class of scalar functions of two variables. The information is the values of f and its partial derivatives. Optimal algorithms in the sense of Sard are studied. Shows the connections with two-dimensional interpolatory splines.

Rivlin, T. J.: See Micchelli, C. A.

Saari, D. G. and Simon, C. P.

"Effective Price Mechanisms,"

Econometrica, 46, 1978, 1097-1125.

core, economic equilibrium, nonlinear equations, multivariate, iterative information, one-point iterations

Considers how much information is required for a price mechanism to converge to an economic equilibrium. Among the results established is the following. Let U be an open subset of R^n and let $f : U \to R^n$ be an excess demand function. Proves that any "local effective price mechanism" requires the evaluation of f and f'. This result follows from Theorem 4.2 in Traub and Woźniakowski [76] "Optimal Linear Information for the Solution of Nonlinear Operator Equations."

Šajdaeva, T. A.,

"Quadrature Formulae with Least Bound for the Remainder for Some Classes of Functions," (in Russian),

Trudy Matem. Instit. im. Steklova, Akad. Nauk SSSR, 53, 1959, 313-341.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for classes of scalar functions with bounded first, second or third derivative. The weights and points of optimal quadrature formulae are derived.

Sard, A.,

"Best Approximate Integration Formulas; Best Approximation Formulas,"

Amer. J. Math., 71, 1949, 80-91.

core, integration, approximation of linear functionals, scalar, optimal linear algorithms

This is probably the first paper which discusses optimal algorithms. It deals with integration for the class of scalar m-times differentiable functions. The information is the values of f at n fixed points t_i. The remainder of a quadrature formula $\sum_{i=1}^{n} f(t_i)k_i$ of order m-1, may be written as $\int_a^b f^{(m)}(t)k(t)dt$ where the kernel k only depends on the weights and points of the quadrature formula. A quadrature formula is called best (now it is called best in the sense of Sard) if $\int_a^b k^2(t)dt$ is minimal with respect to weights k_i. Solves this problem only for small n and m. Extension to approximation of linear functionals is discussed.

Sard, A.,

Linear Approximation,

American Math. Soc., 1963.

mathematics and core, approximation, integration, scalar

Considers, among other problems, integration for a class of scalar functions. The information is the values of f and its derivatives. Optimal quadrature formulae in the sense of Sard are discussed.

Sard, A.,

"Optimal Approximation,"

J. Functional Anal., 1, 1967, 222-244.

core, approximation of linear operators, optimal algorithms

Considers approximation of a linear operator for a class of problems. The information operator is given by n linear operators. Optimal linear algorithms are studied. Generalization of Sard [49]. See also Sard, A., "Optimal Approximation: an Addendum," J. Functional Anal., 2, 1968, 368-369.

Sard, A.,

"Approximation Based on Nonscalar Observations,"

J. Approximation Theory, 8, 1973, 315-334.

core, approximation of linear operators, splines

Considers approximation of a linear operator on a class of elements. The information is a linear operator. Studies spline approximation. This is a continuation of Sard [67].

Sard, A. and Weintraub, S.,

A Book of Splines,

John Wiley and Sons, Inc., New York, 1971.

core, splines

Surveys the theory of spline approximation. Provides optimal approximations of a function, derivatives of a function or its integral. The information is the values of f at regularly spaced points. A Fortran program for spline approximation is given. The cardinal splines are computed for a number of cases. Good bibliography.

Sard, A.: See also Meyers, L. F.

Schmeisser, G.,

"Optimale Quadraturformeln mit semidefiniten Kernen,"

Numer. Math., 20, 1972, 32-53.

core, integration, scalar, optimal algorithms

Considers integration for the class of (2k)-times differentiable scalar functions. The information is the values of f. Assumes that the error of a quadrature formula may be expressed as $c\, f^{(2k)}(\xi)$ and studies the minimization of $|c|$.

Schoenberg, I. J.,

"On Best Approximations of Linear Operators,"

Nederl. Akad. Wetensch., Indag. Math., 67, 1964, 155-163.

core, approximation of linear functionals, optimal linear algorithms, splines

Considers approximation of a linear functional L for a class of scalar m-times differentiable functions. The information is the values of f. Optimal algorithms in the sense of Sard are obtained by operating with L on the interpolatory spline.

Schoenberg, I. J.,

"Spline Interpolation and Best Quadrature Formulae,"

Bull. Amer. Math. Soc., 70, 1964, 143-148.

core, integration, scalar, optimal linear algorithms, splines

Considers integration for a class of scalar m-times differentiable functions. The information is the values of f. Optimal quadrature formulae in the sense of Sard are shown to be the integrals of the interpolatory splines.

Schoenberg, I. J.,

"On Monosplines of Least Deviation and Best Quadrature Formulae,"

SIAM J. Numer. Anal., Ser. B, 2, 1965, 144-170.

core, integration, scalar, optimal linear algorithms, splines

Considers integration for a class of m-times differentiable scalar functions. The information is the values of f and its derivatives. Optimal quadrature formulae in the sense of Sard are studied. Shows the relation between optimal quadrature formulae and monosplines of least deviation from zero. Proves that the classic Hermite and Euler-Maclaurin qudrature formulae are optimal. See also Schoenberg, I. J., "On Monosplines of Least Square Deviation and Best Quadrature Formulae II," SIAM J. Numer. Anal., 3, 1966, 321-328.

Schoenberg, I. J.,

"Monosplines and Quadrature Formulae,"

in Theory and Applications of Spline Functions, edited by T. N. E. Greville, Academic Press, New York, 1969, 157-207.

core, integration, scalar, optimal points of information, optimal linear algorithms, splines

Considers integration for a class of scalar m-times differentiable functions. The information is the values of f and its derivatives. Defines and studies optimal quadrature formulae in the sense of Sard as formulae for which weights and points of information are chosen to minimize $\int_a^b k^2(t)dt$. See Sard [49]. The solution is derived in terms of monosplines of least L_2-norm.

Schoenberg, I. J.,

"A Second Look at Approximate Quadrature Formulae and Spline Interpolation," Advances in Math., 4, 1970, 277-300.

core, integration, scalar, optimal linear algorithms, splines

Considers integration with a weight function for a class of scalar m-times differentiable functions. The information is the values of f and its derivatives at the endpoints of the integration interval. Optimal quadrature formulae in the sense of Sard are studied. Shows that the problem reduces to the monospline of least L_2-norm under some boundary conditions.

Schultz, Martin, H.,

"The Computational Complexity of Elliptic Partial Differential Equations," in Complexity of Computer Computations, edited by R. E. Miller and J. W. Thatcher, Plenum Press, New York, 1972.

core, differential equations, optimal algorithms

Considers an elliptic partial differential equation on a square domain. The information is the values of functions which determine the elliptic equation. Presents a fourth order algorithm on the square root mesh which yields a second order result on the original mesh. This algorithm has combinatory complexity proportional to the number of grid points and therefore is asymptotically optimal.

Schultz, M. H.,

"The Complexity of Linear Approximation Algorithms,"

in Complexity of Computation, edited by R. M. Karp, AMS, 1974, 135-148.

core, approximation, optimal algorithms

Considers approximation for a class of scalar functions. Studies linear algorithms whose range is finite dimensional. Shows that for the class of functions f such that $\|f\| \leq 1$, any linear algorithm has error one. The Kolmogorov n-width serves as a tool to find "good" linear algorithms for the class of absolutely continuous functions whose first derivative is bounded by one in L_∞.

Schultz, M. H.,

"Complexity and Differential Equations,"

in Analytic Computation Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 143-149.

core, approximation, linear and nonlinear equations, multivariate, sparse

Summary of results on three topics: complexity of an approximation theory problem; storage complexity of algorithms for certain classes of sparse linear problems; complexity of two algorithms for solving certain sparse nonlinear systems.

Schumaker, L. L.: See Mangasarian, O. L.

Secrest, D.,
"Numerical Integration of Arbitrarily Spaced Data and Estimation of Errors,"
SIAM J. Numer. Anal., Ser.B, 2, 1964, 52-68.

core, integration, scalar, optimal linear algorithms

Considers integration for the class of scalar functions whose r-th derivative is bounded in L_p by a constant. The information is the values of f. Optimal error quadrature formulae are derived.

Secrest, D.,
"Best Approximate Integration Formulas and Best Error Bounds,"
Math. Comp., 19, 1965, 79-83.

core, integration, scalar, optimal algorithms

Considers integration for the class of scalar functions whose n-th derivative is bounded in L_2 by a constant. The information is the values of f. Optimal algorithms and their errors are derived in terms of natural splines.

Secrest, D.,

"Error Bounds for Interpolation and Differentiation by the Use of Spline Functions,"

SIAM J. Numer. Anal., Ser. B, 2, 1965, 440-447.

core, approximation of linear functionals, optimal error algorithms, error bounds

Considers approximation of a linear functional for the class of scalar functions whose n-th derivative is bounded in L_2 by a constant. The information is the values of f. Optimal error algorithms are derived in terms of natural splines. Optimal error bounds for interpolation and differentiation are presented.

Shamanskii, V. E.,

"A Modification of Newton's Method," (in Russian),

Ukrainskii Matematicheskii Zhurnal, 19, 1967, 133-138.

core, nonlinear equations, multivariate, iterative complexity

Considers a class of algorithms for the solution of a multivariate nonlinear system $f = 0$. Information used are the evaluations of f. A discrete approximation of the Jacobian in Newton iteration is held constant for m_k iterations. The m_k are chosen so as to minimize a complexity index.

These algorithms are a discretized form of algorithms introduced by Traub (see Traub [62] or Traub [64]).

Sharygin, I. F.,

"A Lower Estimate for the Error of Quadrature Formulae for Certain Classes of Functions," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 3, 1963, 370-376.

English translation in U.S.S.R. Computational Math. and Math. Phys., 3, 1963, 489-497.

core, integration, multivariate, lower bounds

Considers integration for three classes of scalar functions of several variables with bounded Fourier coefficients. The information is the values of f. Lower bounds on the error of linear quadrature formulae are given.

Sharygin, I. F.,

"A Lower Bound for the Error of a Formula for Approximation Summation in the Class $E_{s,p}(C)$," (in Russian),

Mathematicheskie Zametki, 21, 1977, 371-375.

English translation in Math. Notes, 21, 1977, 207-210.

core, integration, multivariate, optimal points of information, lower bounds

Considers the integration problem for the class $E_{s,p}(C)$ of scalar functions of s variables. The information is the values of f at p points. It is shown that for any points of information every algorithm has the error at least $\theta(p^{-1} \ln^{s} p)$. This bound is achievable for the optimal algorithm using p equidistant points of information.

Sharygin, I. F.: See also Maung Čžo Njun

Sieveking, M.,

"An Algorithm for Division of Power Series,"

Computing, 10, 1972, 153-156.

core, formal power series, fast algorithms

Gives a fast algorithm for computing the quotient of two formal power series. See also Kung [74].

Simon, C. P.: See Saari, D. G.

Smolyak, S. A.,

"Interpolation and Quadrature Formulas for the Classes W_s^α and E_s^α," (in Russian),

Dokl. Akad. Nauk SSSR, 131, 1960, 1028-1031.

English translation in Soviet. Math. Dokl., 1, 1960, 384-387.

core, interpolation, integration, multivariate, lower bounds

Considers the interpolation and integration problems for two classes W_s^α and E_s^α of complex periodic functions of s variables with bounded Fourier coefficients. The information is the value of f at n points. Lower and upper bounds on the error for optimal points of information are derived. The upper bound is sharp for prime n.

Smolyak, S. A.,

"On Optimal Restoration of Functions and Functionals of Them," (in Russian),

Candidate Dissertation, Moscow State University, 1965.

core, approximation of linear functionals, optimal linear algorithms

This is a Ph.D. thesis not available to us which does not seem to have been published. Considers the approximation of a linear functional for a balanced convex class of functions. The information is the values of n linear functionals. Shows that there exists a linear optimal error algorithm. See Bakhvalov [71] for a proof of this result.

Sobol, I. M.,

Multivariate Quadrature Formulas and Haar Functions, (in Russian),

Nauka, Moscow, 1969.

core, integration, multivariate

Considers the integration problem for scalar functions of several variables belonging to the Fourier-Haar class of functions with bounded coefficients. The information is the values of f. Optimal or close to optimal quadrature formulas are studied. The error is derived.

Sobolev, S. L.,

"On the Order of Convergence of Cubature Formulas" (in Russian),

Dokl. Akad. Nauk SSSR, 162, 1965, 1005-1008.

English translation in Soviet Math. Dokl., 6, 1965, 808-812.

core, integration, multivariate, optimal points of information, optimal linear algorithms, lower, upper bounds

Considers integration for a class of scalar functions of s variables with bounded derivatives up to order r in L_2. The information is the values of f at n points. Proves that the optimal cubature formulae has error roughly $n^{m/s}$.

Sobolev, S. L.,

Introduction to the Theory of Cubature Formulas, (in Russian),

Nauka, Moscow, 1974.

core and mathematics, integration, multivariate, optimal linear algorithms

Considers, among other problems, integration for the class of scalar functions of several variables with bounded r-th derivative in L_2. The information is the values of f. Derives a lower bound for the error of the integration problem. Optimal points of information are studied.

Sobolev, S. L.: See also Babuška, I.

Sonnevend, G.,

"On Optimization of Algorithms for Function Minimization," (in English),

Zh. vychisl. Mat. mat. Fiz., 17, 1977, 591-609.

core, extremum, nonlinear equations, multivariate, optimal points of information, optimal error algorithms

Considers the search for the minimum of scalar functions f of several variables and the search for the zero of systems of nonlinear equations grad $f(x) = 0$ for the class of functions f such that $(\text{grad } f(x)-\text{grad } f(y),x-y)/\|x-y\|^2 \in [m,M]$ where $m \geq 0$. The information is the values of f and its derivative. The optimal points of information and optimal error algorithms with respect to different error criteria are defined. Shows a connection between these problems and dynamic programming and optimal control problems.

Stechkin, S. B.,

"Best Approximation of Linear Operators," (in Russian)

Mathematicheskie Zametki, 1, 1967, 137-148.

English translation in Math. Notes, 1, 1967, 91-99.

core, linear operators, optimal approximation by bounded linear operators

In this paper there is no concept of ϵ-approximation or of information and the definition of optimality is different than in our setting. Considers approximation of an unbounded linear operator U by linear operators φ whose norms do not exceed a given constant. Find optimal operators φ and their errors for several U. If U is a differentiation operator, $Uf = f^{(k)}$ and k is small, the nearly optimal φ requires few evaluations of f.

Stenger, F.,

"Optimal Convergence of Minimum Norm Approximations in H_p,"

Numer. Math. 29, 1978, 345-362.

core, integration, approximation, analytic functions, scalar, optimal linear algorithms, lower, upper bounds

Considers integration and approximation for the class H_p of scalar analytic functions on the unit disc. The information is the values of f at n points. Proves that the minimal error of linear algorithms for both problems is roughly $\exp(-c\sqrt{n})$ where $c > 0$.

Stern, M. D.,

"Optimal Quadrature Formulae,"

The Computer Journal, 9, 1967, 396-403.

core, integration, scalar, optimal linear algorithms

Considers integration for the class of scalar functions whose second derivative is bounded in L_p by a constant. The information is the values of f. Optimal quadrature formulae and their errors are derived.

Stetter, F.,

"On Best Quadrature of Analytic Functions,"

Quart. Appl. Math., 27, 1969, 270-272.

core, integration, analytic functions, scalar, optimal linear algorithms, upper bounds

Considers integration for a class of scalar analytic functions. The information is the values of f. Studies the optimal linear algorithm and derives upper bounds for its error.

Strongin, R. G.,

Numerical Methods for Multivariate Extremal Problems, (in Russian), Nauka, Moscow, 1978.

core, extremum, scalar, optimal Baysian algorithms

Considers the search for the minimum in the class of scalar functions satisfying a probabilistic Lipschitz-like condition. The information is the values of f. Optimal Baysian algorithms are studied.

Sukharev, A. G.,

"Optimal Strategies of the Search for an Extremum," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 11, 1971, 910-924.

English translation in U.S.S.R. Computational Math. and Math. Phys., 11, 1971, 119-137.

core, extremum, multivariate, optimal points of information

Considers the search for the maximum in the class of scalar functions of N variables satisfying a Lipschitz condition with a constant L on the set K. The information is the values of f at n points. Shows that the optimal points of information form the optimal covering of K. The optimal adaptive information is shown to be nonadaptive. For $K = [0,1]^N$, $(L/\epsilon)^N$ function evaluations are necessary to find an ϵ-approximation to the solution. A probabilistic choice of the points of information is also considered.

Sukharev, A. G.,

"Best Sequential Search Strategies for Finding an Extremum," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 12, 1972, 35-50.

English translation in U.S.S.R. Computational Math. and Math. Phys., 12, 1972, 39-59.

core, extremum, multivariate, optimal points of information

This is a continuation of Sukharev [71]. Assumes that the points at which a function is evaluated may depend on the previously computed information. Shows that the optimal points of information form the optimal covering of the domain of f. Related to dynamic programming.

Sukharev, A. G.,

Optimal Search for Extremum, (in Russian),

Moscow State University, 1975.

core, extremum, nonlinear equations, scalar, optimal points of information

Considers the search for the maximum in the class of Lipschitz or unimodal scalar functions. The information is the values of f given simultaneously or adaptively. Surveys many results for this problem. Considers also the solution of nonlinear scalar equations for the class of functions f such that $f(a) \leq 0$, $f(b) \geq 0$ and $(f(x)-f(y))/(x-y) \in [m,M]$ for $x \in [a,b]$ with given positive m,M.

Sukharev, A.G.,

"Optimal Search for a Zero of Function Satisfying Lipschitz's Condition," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 16, 1976, 20-30.

English translation Sukharev, A. G., "Optimal Search for the Roots of a Function Satisfying a Lipschitz Condition," U.S.S.R. Computational Math. and Math. Phys., 16, 1976, 17-26.

core, nonlinear equations, scalar, optimal points of information, optimal error algorithms

Considers the search for a zero in the class of scalar functions satisfying a Lipschitz condition with a given constant on a given interval. The information is the values of f at n points x_i. The optimal choice for the x_i is obtained for the case that f changes sign at the interval end points. If this assumption does not hold, the optimal error algorithm which minimizes the absolute value of f is obtained. In both cases, the algorithms are bisection algorithms.

Sukharev, A.G.,

"Optimal Quadrature Formulas for Some Functional Classes,"

Report, 1978.

core, integration, multivariate, optimal points of information, optimal algorithms

Considers the integration problem for the class of generalized Lipschitz scalar functions of s variables. The information is the values of f at n points. The optimal algorithms and their errors are derived. The optimal points of information are found. The error for optimal points is roughly $\sqrt[s]{n^{-1}}$.

Sukharev, A.G.,

"The Optimal Method for Constructing Best Uniform Approximations for Functions of Certain Class," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 18, 1978, 302-313.

core, approximation, optimal points of information

Considers uniform approximation for the class of scalar functions satisfying a Lipschitz condition with a given constant. The information is the values of f. The optimal points of nonadaptive and adaptive information are discussed.

Taikov, L. V.,

"Kolmogorov-Type Inequalities and the Best Formulas for Numerical Differentiation," (in Russian),

Mathematicheskie Zametki, 4, 1968, 233-238.

English translation in Math. Notes, 4, 1968, 631-634.

core, differentiation, optimal apprixímation by bounded linear operators

Following Stechkin [67], considers approximation of $f^{(k)}$ for the class of scalar functions with bounded n-th derivative in L_2, $0 \leq k < n$, by means of linear operators whose norm is bounded by a given constant. Finds the error of such an optimal approximation.

Tanama, V. P.,

"On the Optimality of Methods of Solving Nonlinear Unstable Problems,"

Dokl. Akad. Nauk SSSR, 220, 1975, 1035-1037.

English translation in Soviet Math. Dokl., 16, 1975, 213-215.

core, operator equations, optimal error algorithms

Considers a nonlinear operator equation $Ax = y$ where A is a one-to-one continuous operator and y belongs to a given set. The information is a perturbed value of Ax. Shows that the "residual principle" algorithm and the "quasi-solution" algorithm are optimal error algorithms to within a factor of two.

Tarassova, V. P.,

"Optimal Strategies of Search for Domain of Greatest Values for Some Classes of Functions," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 18, 1978, 886-896.

core, extremum, optimal error algorithms

Considers the search for the maximum in a class K of functions defined on [a,b] with values in a linear ordered space. The information is the values of f. Assuming that for every $f \in K$ there exists a subinterval $\Delta \subset [a,b]$ of length δ such that $f(x) > f(x')$, $\forall x \in \Delta$, $x' \notin \Delta$, the optimal error algorithms are derived for $\delta = (b-a)/2$.

Tikhomirov, V. M.,

"Best Methods of Approximation and Interpolation of Differentiable Functions in the Space C[-1,1]," (in Russian),

Mat. Sbornik, 80(122), 1969, 290-304.

English translation in Math. U.S.S.R. Sbornik, 9, 1969, 275-289.

mathematics, n-widths

Considers approximation for the class of scalar functions whose (r-1)-th derivatives satisfy a Lipschitz condition with unity. The periodic case is also studied. The Kolmogorov and Gelfand n-widths are expressed in terms of the norms of perfect splines. The extremal subspaces are shown to be spanned by splines.

Tikhomirov, V. M.,

Some Problems of Approximation Theory, (in Russian),

Moscow University, 1976.

mathematics, approximation, n-widths

Considers general problems of approximation. Contains many classical and new results especially for different n-widths (Kolmogorov, Gelfand, linear). Good bibliography.

Tikhomirov, V. M.: See also Kuzovkin, A. I.

Tikhonov, A. N. and Gaisarian, S. S.,

"The Choice of Optimum Networks in the Approximate Calculation of Quadratures," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 9, 1969, 1170-1176.

English translation in U.S.S.R. Computational Math. and Math. Phys., 9, 1969, 252-262.

core, integration, scalar, optimal points of information

Considers quadrature formulae of degree s. The information is the values of f. Optimal points of information are considered. Shows that if $f^{(s)}$ does not change sign on an interval then the unique optimal points exist and are determined by a three term nonlinear recurrence. The connection with suboptimal knots which minimize the dominant error term is shown.

Todd, M. J.,

"Optimal Dissection of Simplices,"

Department of Operations Research Report, Cornell University, 1976.

core, nonlinear equations, multivariate, optimal algorithms

Considers the fixed point problem in a class of continuous functions of several variables. The information is the values of f. Gives nearly optimal triangulations for computing fixed points. Obtains bounds on the asymptotic rate at which the error decreases for optimal algorithms.

Traub, J. F.,

"On Functional Iteration and the Calculation of Roots,"

Preprints of Papers 16th National ACM Conference, Session 5A-1, Los Angeles, CA, 1961, 1-4.

core, nonlinear equations, scalar, one-point iterations, one-point iterations with memory, iterative complexity, maximal order

Initiates research into iterative complexity. Short summary of research later published in Traub [64]. Introduces classification of one-point iteration and one-point iteration with memory (which is called multipoint iteration in this paper). Proves maximal order theorem for one-point iteration. Conjectures memory always adds less than one to order for a one-point iteration with memory.

Traub, J. F.,

"On the Informational Efficiency of Iteration Functions,"

Abstracts of Short Communications, International Congress of Mathematicians, Stockholm, 1962, 202.

core, nonlinear equations, scalar, one-point iterations, one-point iterations with memory, multipoint iterations, maximal order

Considers iterations for computing a simple zero of a scalar nonlinear function. Discusses relation between the information used by an iteration and its maximal order for three classes of iterations: one-point, one-point with memory (which is called modified one-point), and multipoint.

Traub, J. F.,

"Optimal m-Invariant Iteration Functions,"

Notices Amer. Math. Soc., 9, 1962, 122.

core, nonlinear equations, scalar, one-point iterations, maximal order

Considers one-point iterations for solving a scalar nonlinear equation for a zero of known multiplicity m. Seeks a family of iterations of maximal order p for all positive integer m and p. The problem is completely solved; the maximal order iterations have coefficients depending on Stirling numbers of the first and second kind.

Traub, J. F.,

"The Theory of Multipoint Iteration Functions,"

Digest of Technical Papers, 1962, ACM National Conference, Vol. 1, 1962, 80-81.

core, nonlinear equations, scalar, multivariate, multipoint iterations, iterative information, iterative algorithms, order

Introduces concept of multipoint iteration functions for computing a zero of a nonlinear function f. Points out that for multipoint iterations, algorithms of order p do not require the evaluation of the first p-1 derivatives of f. In particular, there exist iterations of order p using p-1 values of f and one value of f and p-1 values of f', and iterations of order 2(p-1) using p-1 values of f, one value of f' and one value of f". See also Shamanskii [67] who has analyzed a discrete version of the first of these iterations.

Traub, J. F.,

"Interpolatively Generated Iteration Functions,"

National ACM Conference, 1963.

core, nonlinear equations, scalar, one-point iterations, one-point iterations with memory, order

Introduces interpolatory iterations for a zero of a scalar nonlinear function. Derives two polynomial equations which determine the order of an interpolatory iteration for a simple or multiple zero.

Traub, J. F.,

Iterative Methods for the Solution of Equations,

Prentice-Hall, Englewood Cliffs, NJ, 1964.

core, nonlinear equations, scalar and multivariate, one-point and multipoint iterations, iterations with memory interpolatory iterations, iterative information, iterative complexity, maximal order

Considers iterations for computing a simple or multiple zero of a scalar nonlinear function f. Also presents some results on multivariate nonlinear functions. The information is the values of f and its derivatives. Some of the results were announced in earlier abstracts and summaries; see papers by Traub from 1961-1963.

Introduces classification of iterations as one-point, one-point with memory, multipoint, multipoint with memory. Proves maximal order theorem for one-point iterations. Introduces idea of interpolatory iteration. Conjectures memory always adds less than one to order for a one-point iteration. Introduces multipoint iteration and shows the maximal order properties of multipoint iteration are very different from one-point iteration. Analyzes computational efficiency. Good bibliography.

Much of the material in this book is contained in a 140 page unpublished manuscript prepared in 1961.

Traub, J. F.,

"Computational Complexity of Iterative Processes,"

SIAM-J. Comput., 1, 1972, 167-179.

core, survey, iterative complexity, maximal order

Surveys research in iterative complexity and gives some history. Introduces terminology analytic computational complexity. Good bibliography.

Traub, J. F.,

"Optimal Iterative Processes, Theorems and Conjectures,"

Information Processing, 71, North-Holland, 1972, 1273-1277.

core, nonlinear equations, scalar, one-point iterations, one-point iterations with memory, multipoint iterations, iterative complexity, maximal order

Surveys iterative complexity as of 1972. Good bibliography.

Traub, J. F.,

"An Introduction to Some Current Research in Numerical Computational Complexity,"

in The Influence of Computing on Mathematical Research and Education, Proceedings of Symposia in Applied Mathematics, Vol. 20, Amer. Math. Soc., Providence, RI, 1974, 47-55.

core, survey, algebraic complexity, analytic complexity, algebraic numbers, parallel algorithms

Surveys some research in algebraic complexity, analytic complexity, and parallel algorithms as of 1974. Good bibliography.

Traub, J. F.,

"Parallel Algorithms and Parallel Computational Complexity,"

Information Processing 74, North-Holland, 1974, 685-687.

core, parallel algorithms, parallel complexity

Surveys some research in parallel algorithms and parallel complexity as of 1974. Defines optimal speed-up for both algebraic and analytic problems. In the methodology of this paper, the speed-up for computing a zero of a nonlinear function is bounded by a constant for any number of processors.

Traub, J. F.,

"Theory of Optimal Algorithms,"

in Software for Numerical Mathematics, edited by D. J. Evans, Academic Press, New York, 1974, 1-13.

core, nonlinear equations, scalar, iterative complexity, algebraic complexity

Surveys iterative complexity and some topics in algebraic complexity as of 1973.

Traub, J. F. (editor),

Analytic Computational Complexity, Academic Press, New York, 1976.

core, iterative information, fast algorithms, iterative complexity, maximal order

Proceedings of a symposium held in 1975. Contains 13 papers on analytic computational complexity.

Traub, J. F.,

"Introduction,"

in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 1-4.

core, analytic complexity

Summarizes some of the reasons for studying analytic complexity. Gives brief overview of 13 invited papers.

Traub, J. F.,

"Recent Results and Open Problems in Analytic Computational Complexity,"

in Mathematical Models and Numerical Methods, Banach Center Publications, Volume 3, PWN, Warsaw, Poland, 1978.

core, nonlinear equations, abstract, iterative complexity, maximal order

Based on a lecture presented at the Banach Center in 1975. Gives a brief survey of research and open problems as of 1975.

Traub, J. F. and Woźniakowski, H.,

"Optimal Linear Information for the Solution of Non-Linear Operator Equations," in Algorithms and Complexity: New Directions and Recent Results, edited by J. F. Traub, Academic Press, New York, 1976, 103-119.

core, nonlinear equations, abstract, optimal linear iterative information, order of information, one-point iterations

Poses a new question: What information is relevant to the solution of a problem? Let f be a nonlinear operator. The information is any finite dimensional linear operator on f. Proves that the maximal order of any one-point iteration using linear information is the cardinality of the information. On the other hand, any iteration of order n using linear information of cardinality n must use the standard information $f(x), f'(x), \ldots, f^{(n-1)}(x)$. That is, any even locally convergent one-point iteration must use the information $f(x), f'(x)$.

Traub, J. F. and Woźniakowski, H,

"Optimal Radius of Convergence of Interpolatory Iterations for Operator Equations,"

Department of Computer Science Report, Carnegie-Mellon University, 1976.

To appear in Aequationes Mathematicae.

core, nonlinear equations, abstract, one-point iterations, radius of convergence

Studies radius of convergence of one-point direct interpolatory iteration as a function of order. Shows for two classes of operator equations that the radius of convergence can be large for large order.

Traub, J. F. and Woźniakowski, H.,

"Strict Lower and Upper Bounds on Iterative Computational Complexity,"

in Analytic Computational Complexity, edited by J. F. Traub, Academic Press,

New York, 1976, 15-34.

core, nonlinear equations, iterative information, one-point iterations,

iterative complexity, complexity index, lower bounds

Studies the complexity of iteration assuming that a simplified error equation holds. Introduces complexity index and shows that complexity is the product of the complexity index and the error coefficient function. Gives strict non-asymptotic lower and upper bounds on complexity. Also gives rigorous conditions for comparing the complexity of two different algorithms.

Traub, J. F. and Woźniakowski, H.,

"Convergence and Complexity of Interpolatory-Newton Iteration in a Banach

Space,"

Department of Computer Science Report, Carnegie-Mellon University, 1977. To

appear in Comp. and Maths with Appls.

core, nonlinear equations, abstract, Newton iteration, optimal iterations,

iterative complexity

The class of Interpolatory-Newton iterations is defined and analyzed for the computation of a simple zero of a nonlinear equation in a Banach space of finite or infinite dimension. Convergence of the class is established. Concepts of "informationally optimal class of algorithms" and "optimal algorithm" are formalized. For the multivariate case, the optimality of Newton iteration is established in the class of one-point iterations under an "equal cost assumption".

Traub, J. F. and Woźniakowski, H.,
"Convergence and Complexity of Newton Iteration for Operator Equations,"
Department of Computer Science Report, Carnegie-Mellon University, 1977.
See also J.ACM, 26, 1979, 250-258.

core, nonlinear equations, abstract, Newton iteration, iterative complexity, optimal convergence

Considers what conditions must be imposed to assure "good complexity" in addition to convergence. Studies Newton iteration for a zero of a nonlinear operator. Establishes optimal radius of ball of convergence with respect to a certain functional. Shows Newton may have arbitrarily high complexity when it converges and conjectures this is a general phenomenon. Establishes radius of ball of good complexity and lower bound on complexity of Newton iteration.

Traub, J. F. and Woźniakowski, H.,
"General Theory of Optimal Error Algorithms and Analytic Complexity, Part A. General Information Model,"
Department of Computer Science Report, Carnegie-Mellon University, 1977.

core, analytic complexity, optimal linear information, optimal error algorithms, central algorithms, linear algorithms, lower bounds

Considers approximation of a linear or nonlinear operator for a class of elements. The information is a linear or nonlinear operator. Studies a general information model of analytic complexity. Introduces the concepts of radius (diameter) of information, optimal linear information operators, optimal error and central algorithms. Among other results, shows the existence of "strongly non-computable" linear problems.

Traub, J. F. and Woźniakowski, H.

Chapter 2 was written jointly with B. Kacewicz,

"General Theory of Optimal Error Algorithms and Analytic Complexity, Part B. Iterative Information Model,"

Department of Computer Science Report, Carnegie-Mellon University, 1978.

core, analytic complexity, nonlinear equations, abstract, iterative information, order of information, iterative algorithms

Considers approximation of a nonlinear operator for a class of elements. The information is a linear "iterative" operator. Studies an iterative information model of analytic complexity. Shows for which problems the class of iterative algorithms using linear one-point information without memory is empty. Conjectures that essentially only nonlinear equations can be solved iteratively.

Traub, J. F. and Woźniakowski, H.,

A General Theory of Optimal Algorithms,

Academic Press, New York, 1980.

core, analytic complexity, general and iterative information, optimal information, order of information, optimal error algorithms, optimal complexity algorithms, linear algorithms, iterative algorithms, lower bounds

This monograph includes considerably extended and improved material from Traub and Woźniakowski [77 and 78], "General Theory of Optimal Error Algorithms and Analytic Complexity, Part A and Part B." It also includes this annotated bibliography.

Traub, J. F.: See also Brent, R. P.;
Feldstein, A.;
Kung, H. T.

Trigiante, D.: See Casuli, V.

Trojan, J. M.,
"Tight Bounds on the Complexity Index of One-Point Iterations," submitted for publication.

core, nonlinear equations, abstract, asymptotically optimal algorithms, one-point iterations, iterative complexity

Considers maximal order one-point iterations for solving nonlinear equation $f = 0$ in a Banach space. The information used by the nth order method is the standard information $f(x), f'(x), \ldots, f^{(n-1)}(x)$. If f is finite dimensional, the combinatory complexity of these methods is linear in the number of pieces of scalar information used. This yields tight bounds on the complexity index. See also J. M. Trojan, "How to Decrease the Combinatory Complexity," Demonstratio Mathematica, 11, 1978, 807-811, where the scalar case is studied.

Tureckij, A. H.: See Aksen, M. B.

Varaiya, P.: See Cohen, A. I.

Velikin, V. L.,

"Optimal Interpolation of Periodic Differentiable Functions with Bounded r-th Derivative," (in Russian),

Mathematicheskie Zametki, 22, 1977, 663-670.

core, interpolation, periodic functions, scalar, optimal points of information

Considers the interpolation problem for the class of periodic scalar functions with bounded r-th derivative in L_∞. The information is the values of f, f' at n points. It is shown that the optimal points of information are equidistant and that 2n function evaluations are more relevant than n function and n first derivative evaluations.

Verner, J. M.: See Cooper, G. J.

Vitushkin, A. G.,

Estimation of the Complexity of the Tabulation Problems, (in Russian), Moscow, Fizmatgiz, 1959.

English translation Vitushkin, A. G., Theory of the Transmission and Processing Information, Pergamon Press, 1961.

core, optimal coding, entropy

Considers the optimal coding for a given class F of functions defined on a domain G. Suppose that for a fixed $\epsilon > 0$, there exist a finite set w and a function $P_k = P_k(x,y)$, $y = (y_1,y_2,\ldots,y_p)$ which is a polynomial of degree at most k with respect to each y_i, $i = 1,2,\ldots,p$, such that for every function $f \in F$ there exists $y(f) \in w^p$ such that $|f(x)-P_k(x,y(f))| \leq \epsilon$, $\forall x \in G$. Then $T_\epsilon^\Phi(f) = \{y(f),P_k\}$ is called the table of f where Φ is a space containing f and $P_k(\cdot,y)$. If $n = \mathrm{card}(w)$ is the total number of elements of w then n^p is the total number of different elements which can occur to code all functions from F. $P(T_\epsilon^\Phi(F)) = \log_2 n^p$ is called the size (or complexity) of the table and measures the number of binary bits necessary to represent n^p elements. Shows that for optimal coding $P(T_\epsilon^\Phi(F))$ is essentially equal to the ϵ-entropy $H_\epsilon(F)$ of F. For some sets F, the basic sharp inequality

$$p \log_2 \left(\frac{k+1}{\epsilon}\right) \geq c(F)H_\epsilon(F)$$

is proven ($c(F)$ is a positive constant). Finds ϵ-entropy for many important classes of functions.

Wacker, H. J.,
"Minimierung des Rechenaufwandes bes Globalisierungen Spezieller Iterations-verfahren von Typ Minimales Residuum,"
Computing 18, 1974, 209-224.

core, nonlinear equations, abstract

Considers the solution of a nonlinear equation $T(x) = 0$ in a Hilbert space. The continuation method is employed, and $T(x,s_i) = 0$ with $0 = s_0 < s_1 < \ldots < s_b = 1$ where $T(x,1) \equiv T(x)$ is solved by a locally convergent iteration starting with the computed approximation x_{i-1} to the solution of $T(x,s_{i-1}) = 0$. The optimal equidistant points s_i are considered.

Walsh, J. L.: See Ahlberg, J. H.

Wasilkowski, G.W.,
"N-Evaluation Conjecture for Multipoint Iterations for the Solution of Scalar Nonlinear Equations,"
Master's thesis, Department of Math., University of Warsaw, 1977. To appear in J.ACM.

core, nonlinear equations, scalar, order of information, multipoint iterations without memory

Considers multipoint iterations without memory for the solution of nonlinear scalar equations. The information is N values of f and its derivatives generated by an incidence matrix. Proves that the order of such iterations is no higher than 2^{N-1} whenever the corresponding Birkhoff interpolation problem is well-poised in the complex case.

Wasilkowski, G. W.,

"Can Any Stationary Iteration Using Linear Information be Globally Convergent?,"

Department of Computer Science Report, Carnegie-Mellon University, 1978.

To appear in J.ACM.

core, nonlinear equations, scalar, iterative information, one point iterations, multipoint iterations, global convergence

Proves that a stationary iteration which uses linear information cannot be globally convergent. Shows this result holds even for as simple a class of problems as the set of all analytic complex functions having only simple zeros. Conjectures the result holds even for the class of all real polynomials with real simple zeros.

Wasilkowski, G. W.,

"Any Iteration for Polynomial Equations Using Linear Information has Infinite Complexity,"

Department of Computer Science Report, Carnegie-Mellon University, 1979.

core, nonlinear equations, scalar, linear information, complexity

Considers the iterative solution of polynomial equations with simple zeros. The information is linear and adaptive. Proves that for any iteration φ and any number k, there exists a polynomial f with all simple zeros such that the first k approximations produced by φ do not approximate a zero α of f better than a starting approximation x_0. This holds even if $|x_0-\alpha|$ is arbitrarily small. This result implies that complexity of any iteration is infinite for the class of polynomial equations with simple zeros.

Wasilkowski, G. W.,

"The Strength of Nonstationary Iteration,"

Department of Computer Science Report, Carnegie-Mellon University, 1979.

core, nonlinear equations, abstract, linear information, global convergence

Considers the iterative solution of nonlinear equations in a Banach space. The information is linear and adaptive. Proves the existence of globally convergent nonstationary iterations for the class of analytic operators with simple zeros.

Wasilkowski, G. W. and Woźniakowski, H.,

"Optimality of Spline Algorithms,"

Department of Computer Science Report, Carnegie-Mellon University, 1978.

core, approximation of linear operators, spline algorithms, optimal linear algorithms, central algorithms

Considers approximation of a linear operator for a class of elements. The information is a linear finite dimensional operator. Introduces the concept of spline algorithms and establishes optimality properties of these algorithms. This unifies and generalizes many known results.

Weinberger, H. F.,

"Optimal Approximation for Functions Prescribed at Equally Spaced Points,"

J. Res. Nat. Bur. Standards, Sect. B, 65, 1961, 99-104.

core, approximation of linear functionals, optimal error algorithms

Considers approximation of a linear functional for the class of scalar functions whose kth derivative is bounded in L_2 by a constant. The information is the values of f at n equidistant points. Optimal error algorithms and their errors are derived. The computation of the optimal algorithm involves the inversion of a matrix of size k-1.

Weinberger, H. F.,

"On Optimal Numerical Solution of Partial Differential Equations,"

SIAM J. Numer. Anal., 9, 1972, 182-198.

core, approximation of linear operators, differential equations, optimal algorithms

Considers approximation of a linear operator $S : B \to \Sigma$ for the unit ball in B. Information is the value of a linear operator $\mathfrak{N} : B \to R^n$. For a given linear operator $M : R^m \to \Sigma$, the operator S is approximated by MQN where Q is a nxm matrix. For fixed n and m, the optimal choice of Q, M and N is studied. The optimal error is expressed in terms of the norms of S and S^*. The results are illustrated for parabolic and elliptic differential equations.

Weinberger, H. F.: See also Golomb, M.

Weintraub, S.: See Sard, A.

Werner, M.: See Barrar, R. B.; Loeb, H. L.

Werschulz, A. G.,
"Computational Complexity of One-Step Methods for Systems of Differential Equations,"
Department of Computer Science Report, Carnegie-Mellon University, 1976.
To appear in Math. Comp.

core, ordinary differential equations, optimal order

This is a part of a Ph.D. Thesis. Considers the solution of an initial value problem for a system of N ordinary differential equations. The information is the values of the right-hand side function and its derivatives. Exhibits an algorithm for the p-th order Taylor method with $\Theta(p^N \log p)$ combinatory complexity. Finds an optimal order which minimizes the complexity bounds of obtaining an ϵ-approximation and shows that under reasonable hypotheses the order tends to infinity as ϵ goes to zero.

Werschulz, A. G.,

"Computational Complexity of One-Step Methods for the Numerical Solution of Initial Value Problems,"

Department of Computer Science Report, Carnegie-Mellon University, 1976.

To appear in Computing.

core, ordinary differential equation, optimal order, optimal step-size

This is a part of a Ph.D. Thesis. Considers the solution of a class of ordinary differential equations. The information is the values of the right-hand side function. Studies the optimal order and optimal step-size of one-step methods which minimize the complexity of obtaining an ϵ-approximation. Shows that the optimal order increases monotonically and tends to infinity as ϵ tends to zero.

Werschulz, A. G.,

"Optimal Order and Minimal Complexity of One-Step Methods for Initial Value Problems,"

Department of Computer Science Report, Carnegie-Mellon University, 1976.

core, ordinary differential equations, optimal order.

This is a part of a Ph.D. Thesis. Considers the solution of an initial value problem in a class of ordinary differential equations. The information is the values of the right-hand side function. Finds an optimal order of one-step methods which minimizes the complexity of obtaining an ϵ-approximation. Shows that under reasonable hypotheses the optimal order tends to infinity as ϵ goes to zero.

Werschulz, A. G.,

"Maximal Order and Order of Information for Numerical Quadrature,"

Mathematics Research Report 77-2, University of Maryland Baltimore County, 1977. See also J.ACM, 26, 1979, 527-537.

core, integration, scalar, order of information, maximal order

Considers integration for the class of analytic functions. The information is the values of f and its derivatives. Defines the concepts of local order and order of information. Studies methods which use fixed information and have maximal order. Proves that the maximal order is equal to the order of information. Finds the order of information for "equally-weighted" Hermitian information.

Werschulz, A. G.,

"Maximal Order for Approximation of Derivatives,"

Mathematics Research Report 77-8, University of Maryland Baltimore County, 1977. To appear in System Sciences.

core, differentiation, order of information, maximal order

Considers differentiation for the class of smooth scalar functions. The information is the values of f. Defines the concept of order of a method and order of information. Shows that the maximal order of methods using fixed information is equal to the order of information. Proves that the central difference formula has maximal order.

Werschulz, A. G.,

"Maximal Order for Quadratures Using n Evaluations,"

Mathematics Research Report 77-7, University of Maryland Baltimore County, 1977. To appear in Aequationes Mathematicae.

core, integration, scalar, maximal order.

Considers integration for the class of analytic functions. The information is the values of n functionals L_i of f where $L_i(f) = f^{(j_i)}(x_i)$, $i = 1,2,\dots,n$, for some j_i. Conjectures that the order of information (see Werschulz [77, Report 77-2]) is at most 2n+1. Proves this conjecture for hermitian information.

Werschulz, A. G.,

"Maximal Order and Order of Information for Local and Global Numerical Problems,"

Proceedings of the 1978 Conference on Information Sciences and Systems, John Hopkins University, 1978.

core, discretized information, order of information, optimal algorithms

Considers approximation of S(f;h) for f from a given set when a real h tends to zero. The information is an operator $\mathfrak{N}(f,h)$. Defines the order of information and proves that this is the sharp upper bound on the orders of algorithms which use $\mathfrak{N}(f,h)$. Several examples illustrate the paper.

Werschulz, A. G.,

"Multipoint Methods with Memory Using Hermitian Information,"

Proceedings of the 1979 Conference on Information Sciences and Systems,

Johns Hopkins University, 1979.

core, nonlinear equations, scalar, hermitian information, memory, maximal order

Considers multipoint iterations with memory for the solution of nonlinear scalar equations. The information is the values of f and its derivatives. Studies maximal order of such iterations and shows that 2^n is the sharp bound on the maximal order.

Wilde, D. J.,

Optimum Seeking Methods,

Prentice-Hall, Englewood Cliffs, NJ, 1964.

core, extremum, scalar, optimal algorithms

A text on the search for the maximum of a function. The information is the values of f. Considers, among others, Fibonaccian search for the class of unimodal functions.

Wilde, D. J.: See also Avriel, M.; Beamer, J. H.

Wilf, H. S.,

"Exactness Conditions in Numerical Quadrature,"

Numer. Math., 6, 1964, 315-319.

core, integration, scalar, optimal points of information, optimal algorithms

Considers integration for a class of scalar analytic functions on the unit disc in a Hilbert space. The information is the values of f. Optimal weights and points of quadrature formulae are considered.

Winograd, S.,

"Parallel Iteration Methods,"

in Complexity of Computer Computations, edited by R. E. Miller and J. W. Thatcher, Plenum Press, New York, 1972, 53-60.

core, nonlinear equations, scalar, iterations with memory, parallel complexity, iterative complexity, maximal order

Considers solution of a scalar nonlinear equation f = 0 on a parallel computer. Considers a class of iterations for which the information is f and its derivatives. Shows that for the complexity model of this paper the speed-up is logarithmic in the number of processors.

Winograd, S.,

"Some Remarks on Proof Techniques in Analytic Complexity,"

in Analytic Computational Complexity, edited by J. F. Traub, Academic Press,

New York, 1976, 5-14.

core, lower bounds

Discusses "fooling" or "adversary" argument to obtain lower bounds in analytic complexity. For illustration shows how this argument is used to obtain lower bounds on algorithms for the maximum of a unimodal function, zero of a scalar function, solution of an integral equation, and approximation of a scalar function.

Winograd, S.: See also Brent, R. P.; Micchelli, C. A.

Wixom, J. A.: See Barnhill, R. E.

Wolfe: See Brent, R. P.

Woźniakowski, H.,

"On Nonlinear Iterative Processes in Numerical Methods," (in Polish),

Ph.D. Thesis, University of Warsaw, 1972.

core, nonlinear equations, abstract, one-point iterations, iterations with memory, interpolatory iterations, maximal order

Considers iterative solution of nonlinear equations. The information is the values of f and its derivatives. Studies maximal order one-point iterations without and with memory. Generalizes interpolatory iterations with memory to multivariate cases. See also Woźniakowski [74].

Woźniakowski, H.,
"Maximal Stationary Iterative Methods for the Solution of Operator Equations,"
SIAM J. Numer. Anal., 11, 1974, 934-949.

core, nonlinear equations, abstract, one-point iterations, iterations with memory, interpolatory iterations, maximal order

Generalizes problem of maximal order to infinite dimensional problems. Establishes maximal order of interpolatory algorithms in scalar case. Shows that memory does not in general increase order in multivariate case.

Woźniakowski, H.,
"Generalized Information and Maximal Order of Iteration for Operator Equations,"
SIAM J. Numer. Anal., 12, 1975, 121-135.

core, nonlinear equations, abstract, order of information, one-point iterations, iterations with memory, interpolatory iterations

Introduces concept of order of information which provides general tool for establishing maximal order of an algorithm. Shows maximal order depends only on information used by an algorithm and not on the structure of the algorithm. Proves that any generalized interpolatory algorithm has maximal order.

Woźniakowski, H.,

"Properties of Maximal Order Methods for the Solution of Nonlinear Equations,"

ZAMM, 55, 1975, 268-271.

core, nonlinear equations, abstract, Hermitian information, order of information, multipoint iterations for scalar problems, iterative complexity

Studies properties of maximal order iterations. Announces that the Kung and Traub conjecture holds for $n \leq 3$ and for "Hermitian" information.

Woźniakowski, H.,

"Maximal Order of Multipoint Iterations Using n Evaluations,"

in Analtyic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 75-107.

core, nonlinear equations, scalar, Hermitian information, iterative information, order of information, multipoint iterations

Studies maximal order of multipoint iteration for solution of the scalar equation $f^{(m)} = 0$, $m \geq 0$. Information is n evaluations of f or its derivatives. Let $p_n(m)$ denote the maximal order. Establishes for Hermitian information the Kung and Traub conjecture that $p_n(0) = 2^{n-1}$. Conjectures that $p_n(m) = 2^{n-1}$. Shows relation between the problems of maximal order and Birkhoff interpolation.

Woźniakowski: See also Jankowski, M.;
Kacewicz, B.;
Traub, J. F.;
Wasilkowski, G. W.

Yun, David Y. Y.,
"Hensel Meets Newton--Algebraic Constructions in an Analytic Setting,"
in Analytic Computational Complexity, edited by J. F. Traub, Academic Press, New York, 1976, 205-215.

core, Newton iteration, algebraic complexity

Discusses use of Newton iteration (which is usually considered an analytic technique) for the solution of algebraic problems such as p-adic approximation.

Zaliznyak, N. F. and Ligun, A. A.,
"On Optimum Strategy in Search of Global Maximum of Function," (in Russian),
Zh. vychisl. Mat. mat. Fiz., 18, 1978, 314-321.

core, extremum, scalar, optimal linear information, optimal error algorithms

Considers the search for the maximum in a class which is the algebraic sum of a convex, compact and balanced set and a finite dimensional linear space. The information is the values of n linear functionals. The optimal error algorithms are derived. The optimal adaptive information is shown to be non-adaptive. For the class of 2π periodic functions whose rth derivative is bounded in L_∞ by unity, the optimal information is the values of f at n equidistant points, the optimal error algorithms are related to splines, and the error is the n-widths of the problem, which is equal to K_r/n^r where K_r is the Favard constant.

Žensykbaev, A. A.,

"On the Best Quadrature Formula on the Class $W^r L_p$," (in Russian),

Dokl. Akad. Nauk SSSR, 227, 1976, 277-279.

English translation in Soviet Math. Dokl., 17, 1976, 377-380.

core, integration, periodic functions, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar periodic functions with bounded r-th derivative in L_p. The information is the values of f. Proves that the rectangle quadrature formula with equidistant points is an optimal linear algorithm with optimally chosen points of information. Generalizes Motornyi [73] where this result was proven for $p = \infty$.

Žensykbaev, A. A.,

"Best Quadrature Formulas for Some Classes of Nonperiodic Functions," (in Russian),

Dokl. Akad. Nauk SSSR, 236, 1977, 531-534.

English translation in Soviet Math. Dokl., 18, 1977, 1222-1226.

core, integration, scalar, optimal points of information, optimal linear algorithms

Considers integration for the class of scalar functions with bounded r-th derivative in L_p. The information is the values of f. Optimal linear algorithms with optimal points of information and their errors are studied.

Žensykbaev, A. A.,

"On a Property of the Best Quadrature Formulae," (in Russian),

Matematicheskie Zametki, 23, 1978, 551-562.

core, integration, scalar, optimal linear algorithms

Considers integration for the class of scalar functions whose r-th derivative is bounded in L_q. The information is the values of $f, f', \ldots, f^{(\zeta)}$. Shows that the optimal quadrature formulae coincide for $\zeta = 2m$ and $\zeta = 2m+1$. This also holds for the periodic case.

Zhileikin, Ya. M. and Kukarkin, A. B.,

"On the Optimal Evaluation of Integrals with Strongly Oscillating Integrand," (in Russian),

Zh. vychisl. Mat. mat. Fiz., 18, 1978, 294-301.

core, integration, scalar, optimal error algorithms

Considers the approximation of $\int_0^1 e^{iwg(x)} f(x)\, dx$ for a fixed function g, large $|w|$, and for the class of functions f with bounded rth derivative. The information is the values of f. The optimal error algorithms and their errors are presented for $g \in C^{r+1}$.

Zhilinskas, A. G.,

"One-Step Baysian Method for Searching for the Extremum of Functions of One Variable," (in Russian),

Cybernetics, Academy of Sciences of Ukrainian of USSR, 1, 1975, 139-144.

core, extremum, scalar, optimal Baysian algorithms

Considers the search for the minimum in a class of scalar functions related to the Wiener process. Presents an optimal one-step Baysian algorithm.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CMU-CS-79-140	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ANNOTATED BIBLIOGRAPHY AND BRIEF HISTORY OF OPTIMAL ALGORITHMS AND ANALYTIC COMPLEXITY		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. F. TRAUB H. WOZNIAKOWSKI		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0370
9. PERFORMING ORGANIZATION NAME AND ADDRESS Carnegie-Mellon University Computer Science Department Pittsburgh, PA 15213		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA 22217		12. REPORT DATE August 1979
		13. NUMBER OF PAGES 172
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)